Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Optimizing Reorder Intervals for Two-Echelon Distribution Systems with Stochastic Demand

Kevin H. Shang, Zhijie Tao, Sean X. Zhou

Operations Research
Articles in Advance, pp. 1–18
ISSN 0030-364X (print) | ISSN 1526-5463 (online)

http://dx.doi.org/10.1287/opre.2015.1347

Optimizing Reorder Intervals for Two-Echelon Distribution Systems with Stochastic Demand

Kevin H. Shang
Fuqua School of Business, Duke University, Durham, North Carolina 27708, khshang@duke.edu

Zhijie Tao
School of International Business Administration, Shanghai University of Finance and Economics, Shanghai 200000, China, tao.zhijie@mail.shufe.edu.cn

Sean X. Zhou
Department of Decision Sciences and Managerial Economics, CUHK Business School, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, zhoux@baf.cuhk.edu.hk

We consider a periodic-review inventory system in which N non-identical retailers replenish from a warehouse, which further replenishes from an outside vendor with ample supply. Each facility faces Poisson demand and replenishes according to a base-stock policy in a fixed time interval. Fixed costs are incurred for placing an order. The warehouse fills the retailers' orders in the same sequence as the occurrence of the demand at the retailers. The objective is to minimize the average system cost per period. This paper develops an evaluation scheme and provides a method to obtain the optimal base-stock levels and reorder intervals. Specifically, with fixed reorder intervals, we show that the optimal base-stock levels can be obtained by generalizing the result in the literature. To find the optimal reorder intervals, we first allocate the total system cost to each facility and then construct a lower bound to the allocated facility cost. These lower bound functions, which are separable functions of reorder intervals, can be used to derive bounds for the optimal reorder intervals. The key to tightening the bounds is to obtain a near-optimal total cost. Thus, we propose a simple heuristic that modifies the algorithm that solves the deterministic counterpart. The results of numerical studies suggest that the optimal reorder intervals tend to satisfy integer-ratio relationships and that the suggested heuristic can generate effective integer-ratio policies for large systems.

Subject classifications: multi-echelon; inventory/production; stochastic; approximations/heuristics.
Area of review: Operations and Supply Chains.
History: Received May 2011; revisions received June, 2013, April, 2014; accepted July 2014. Published online in *Articles in Advance*.

1. Introduction

We consider a one-warehouse-multi-retailer system (OWMR) in which each facility replenishes inventory in a fixed time interval. There are N nonidentical retailers, each facing independent Poisson demands. The retailers replenish their inventory from the warehouse, which further replenishes from an outside supplier with unlimited stock. Unfilled demand is fully backlogged. Let j be the facility index, where the warehouse is indexed as 0 and the retailers $j = 1, \ldots, N$. Each facility implements an echelon (S, T) policy: Facility j reviews its echelon order inventory position (= inventory on order + inventory on hand + inventory at or in transit to the downstream facilities − total backorders at the retailers) every T_j time units and orders up to a base-stock level S_j. Under this policy, the system repeats its ordering pattern with the cycle length being the least common multiplier of the reorder intervals of all facilities. We assume that the order schedules are *synchronized*. More specifically, when the warehouse receives a shipment at the beginning of a system's order cycle, all retailers place an order. Consequently, shipments to the facilities occur on multiples of the facility's reorder intervals. The warehouse applies the so-called virtual allocation rule (e.g., Axsäter 1993, Graves 1996). That is, whenever a unit is demanded at one of the retailers, the warehouse assigns one unit in stock or on order to this retailer. At a replenishment epoch for this retailer, all assigned inventory units at the warehouse are shipped to this retailer.[1] A linear echelon holding cost h_j is incurred per period and per unit carried at facility j ($\geqslant 0$) and a backorder cost b_j is incurred per period and per unit backlogged at retailer j ($\geqslant 1$). In addition, a fixed order cost K_j is incurred at facility j ($\geqslant 0$) for placing an order. The objective is to obtain policy parameters S_j and T_j such that the average total cost per period is minimized.

This type of periodic replenishment scheme is commonly implemented in practice because it is simple to plan labor, coordinate production, and deliver materials according to a fixed schedule. For its deterministic demand counterpart, although it is not clear how to find the optimal reorder intervals, the literature has shown that this periodic replenishment pattern is very efficient in terms of minimizing fixed order and holding costs. Specifically, there exists a very simple integer-ratio policy that guarantees 94% effectiveness. (Let $\mathbb{N}$ denote the set of positive integers and T_b be some base period. An integer-ratio policy satisfies the following conditions: $T_j = n_j T_b$, $n_j \in \mathbb{N}$ for all j and either T_j/T_0 or $T_0/T_j \in \mathbb{N}$ for $j = 1, \ldots, N$.) If the base period T_b can be chosen as a noninteger value, there exists a power-of-two (POT) policy that guarantees 98% effectiveness (Roundy 1985). (A POT policy is a subset of integer-ratio policies where $T_j = 2^{k_j} T_b$, $k_j \in \mathbb{N}$ for all j.) Although the reorder intervals generated from the deterministic model have been used as approximations for the stochastic model (e.g., Chen and Zheng 1997), it is not clear whether such an approximated solution is effective.

This study aims to answer three fundamental questions. First, how can one obtain optimal (S, T) parameters? Liu and Song (2012) recently revisited the single (S, T) model in Rao (2003) and indicated that solving the single-stage problem is more complicated than the algorithm suggested by Rao because of nonconvexity of the total cost function. With this finding, it is conceivable that solving the optimal policy for the OWMR model would be extremely difficult. Second, what properties do the optimal reorder intervals possess? Is the class of integer-ratio policies a good candidate for designing a heuristic policy? Third, how effective is the POT policy obtained from Roundy's algorithm when applying it to our stochastic model? Is there a simple heuristic that outperforms the deterministic POT policy in general?

Since it is not clear whether an integer-ratio policy is optimal, we do not impose any restrictions on the reorder intervals except that they are integers. We first characterize the dynamics of the key inventory variables and provide a bottom-up recursion to evaluate the average system cost. We further show that with fixed reorder intervals, the optimal echelon base-stock levels can be obtained by generalizing a result by Axsäter (1990) (see Appendix A). Our main contribution in this paper is to provide a method for obtaining the optimal reorder intervals. This is achieved by the following steps. First, we decompose the original system and allocate the total system cost to the facilities. Second, we construct a lower bound to the allocated facility cost. These lower bounds are a function of each facility's reorder interval and thus are independent of each other. Third, we propose a simple and effective heuristic based on these lower bound functions. The heuristic cost as well as the lower bound functions generate bounds for the optimal reorder intervals. Consequently, the optimal reorder intervals can be found by enumerating all solutions within the bounds. To reduce the computational effort, we further prove a property that may reduce the number of enumerations: if the reorder intervals of the facilities satisfy integer-ratio relationships, the reorder interval of the warehouse must be no shorter than the minimum of those of the retailers. This result corresponds to the "last-minute property" (the warehouse ships only when at least one retailer orders) established by Schwarz (1973) for the deterministic OWMR model.

A key element of finding tighter parameter bounds is to obtain an effective heuristic cost. Through an extensive numerical study, we find that almost all of the optimal solutions are integer-ratio policies. This motivates us to focus on this class when proposing a heuristic. We first examine the POT policy generated from Roundy's algorithm. To our surprise, it could lead to significant suboptimality, especially when $K_0/(h_0\lambda_0)$ is relatively smaller than $K_j/(h_j\lambda_j)$, where λ_j is the demand rate for the retailer j, and $\lambda_0 = \sum_{i=1}^{N} \lambda_i$. Under this condition, Roundy's algorithm often yields a smaller reorder interval for the warehouse, whereas the optimal reorder intervals tend to be equal among all facilities. There are two reasons for this observation. First, the deterministic model does not consider the demand variability, so the resulting total cost function may not be close to that of the stochastic model. Second, the deterministic model cannot reflect the benefit of risk pooling at the warehouse. That is, in the optimal solution, the warehouse tends to use a larger reorder interval to consolidate the retailers' orders so as to take advantage of risk pooling. To mitigate the impact of these inefficiencies, we propose a heuristic that revises the clustering step (i.e., identifying facilities that use the same reorder interval) in Roundy's algorithm by considering the derived lower bound functions. At the end of the procedure, we generate an effective integer-ratio policy. A numerical study suggests that the heuristic is near optimal: the average optimality gap is 0.4%. To facilitate the implementation and shipment coordination, firms may want to restrict the reorder intervals to be POT integers. Our heuristic approach can easily be revised to generate effective POT reorder intervals. We refer the reader to §5 for a detailed summary of the effectiveness of the heuristic and the quality of the parameter bounds.

We provide a literature review, focusing on papers that employ replenishment strategies with constant replenishment intervals at every facility. Graves (1996) considered a general distribution system in which inventory is controlled by a local (S, T) policy. Graves found that most of the safety stock should be held at the retailer sites and that the virtual allocation rule is near optimal in his numerical study. Axsäter (1993) studied a special case of Graves' model in that the retailers have identical reorder intervals and the order schedule is nested; i.e., $T_0/T_j \in \mathbb{N}$ for $j = 1, \ldots, N$. He demonstrated that a local (S, T) policy under the virtual allocation rule in Graves (1996) is essentially the same as an echelon (S, T) policy. Çetinkaya and Lee (2000) considered a supplier that orders according to an (s, S) policy and ships to the retailers in a fixed time interval. They evaluated the system cost and provided a method to find the optimal inventory policy and the shipping schedule for the supplier. Gürbüz et al. (2007) considered a centralized distribution system consisting of one cross-docking warehouse (no inventory held) and N identical retailers. They proposed a new policy, referred to as the hybrid policy, under which the warehouse monitors the inventory position at all retailers and places an order to raise every retailer's inventory position to an order-up-to level S whenever any retailer's inventory position hits s or the total demand at all retailers reaches Q. They provided a method to optimize the hybrid policy. Chu and Shen (2010) considered a distribution system with a prespecified service level at each location. To satisfy the service levels, safety stocks are kept at the warehouse and the retailer sites in order to buffer random normal demand. They provided an algorithm to find effective POT reorder intervals. Marklund (2011) considered a continuous review system in which the warehouse implements a local (r, Q) policy and allocates stock to the retailers according to the virtual allocation rule. The retailers implement an (S, T) policy. Marklund assumed that the batch size at the warehouse is fixed (i.e., no fixed cost consideration at the warehouse). He provided a method to evaluate the average system cost and proposed heuristics for the policy parameters.

There is a stream of research on the distribution model aiming to study the impact of order schedules on the bullwhip effect. Noteworthy examples include Lee et al. (1997), Cachon (1999), Chen and Samroengraja (2004), and Cheung and Zhang (2008). Finally, our model is a generalization of the stochastic joint replenishment problem (JRP) studied by Atkins and Iyogun (1988), who showed that the periodic review (S, T) policy outperforms the can-order policy (Silver 1981, Federgruen et al. 1984) when the major fixed cost is large. The optimization method developed in this paper can be used for solving the JRP in Atkins and Iyogun (1988).

The rest of the paper is organized as follows. Section 2 introduces the model and the notation. Section 3 characterizes the inventory variables and evaluates the total cost per period. Section 4 provides an approach for finding the optimal reorder intervals. Section 5 conducts a numerical study to examine the optimal solution and the effectiveness of the deterministic POT policy and the proposed heuristic. Section 6 summarizes our major conclusions. Appendix A shows how to optimize the base-stock levels when the reorder intervals are fixed. Appendix B provides proofs. Appendix C presents the optimal solution, heuristic solution, as well as the parameter bounds for the numerical instances we tested in this paper.

2. The Model

We consider a periodic-review, two-echelon distribution system in which a single warehouse supplies N nonidentical retailers. Time periods are indexed as $0, 1, 2, \ldots$. Retailer j faces Poisson demands with stationary rate λ_j. The demands are independent among retailers. Let $[t, t+\tau)$ and $[t, t+\tau]$ denote the time interval over periods $t, t+1, \ldots, t+\tau-1$ and periods $t, t+1, \ldots, t+\tau$, respectively. Let $D_j[t, t+\tau)$ and $D_j[t, t+\tau]$ denote the cumulative demand over time periods in $[t, t+\tau)$ and $[t, t+\tau]$, respectively. There is constant lead time L_j for facility j, $j = 0, \ldots, N$. Let $L_{[0,j]} = L_0 + L_j$. Each facility implements a stationary echelon (S, T) policy. More specifically, retailer j orders up to a base-stock level S_j at the beginning of every T_j period if its inventory order position (inventory on order + inventory on hand − backorders) is less than S_j. Similarly, warehouse orders up to an echelon base-stock level S_0 at the beginning of every T_0 periods if its echelon inventory order position (inventory on order + inventory on hand + inventory at or in transit to the retailers − total backorders at the retailers) is less than S_0. We refer to these T_j-th periods as *order periods* and the moment of placing an order as *order epoch*. We assume that the reorder intervals are positive integers. Let h_j be the echelon holding cost rate for facility j, $j = 0, \ldots, N$, and the local holding cost rate $H_j = h_0 + h_j$ for $j = 1, \ldots, N$. Unmet demand is fully backlogged at each retailer. Let b_j be the backorder cost rate for retailer j, $j = 1, \ldots, N$. Finally, there is a fixed cost K_j associated with each order placed at facility j, $j = 0, \ldots, N$. The objective is to determine (S_j, T_j), $j \geqslant 0$, such that the average total system cost per period is minimized.

Under the echelon (S, T) policy, the system will repeat its order pattern with the cycle length of M periods, where

$$M = \operatorname{lcm}\{T_0, T_1, \ldots, T_N\}.$$

Here, lcm is an operator that generates the least common multiplier of a set of positive integers, e.g., $\operatorname{lcm}(2, 3, 4) = 12$. We assume that the ordering activities between the warehouse and the retailers are coordinated in a *synchronized* manner: When the warehouse receives a shipment at the beginning of a system's order cycle, all retailers place an order. For example, consider a one-warehouse-two-retailer system with $L_0 = 3$, $T_0 = 2$, $T_1 = 1$, and $T_2 = 3$. Suppose that the warehouse places an order at the beginning of period $t, t+2, t+4, \ldots$. An order placed at t will arrive at the beginning of period $t + L_0 = t + 3$. This is the moment that both retailers place an order. Thus, the order periods for retailer 1 are $t+3, t+4, t+5, \ldots$ and for retailer 2 are $t+3, t+6, t+9, \ldots$. We term such an order coordination *synchronized ordering*. In other words, if we define t and $t + L_0$ as the starting time of a cycle for the warehouse and the retailers, respectively, the next cycle will start in M periods later under the synchronized ordering rule.

One concern with synchronized ordering rule is that it may lead to a bigger bullwhip effect when the system is controlled by local policies (Lee et al. 1997). However, Cheung and Zhang (2008) pointed out that a high bullwhip effect may not necessarily lead to a high system cost. Moreover, the bullwhip effect has less impact in our model because demand is learned by the warehouse when it occurs (i.e., echelon control).

Under an echelon (S, T) policy, the retailer fills incoming demand as if it were a single-stage system. That is, when a unit of demand arrives, the retailer fills the demand and the retailer's net inventory level (= inventory on hand minus backorders) is reduced by one unit. The retailer does not place an order until its next order epoch. On the other hand, the warehouse immediately learns about this arriving demand and commits one unit of on-hand inventory (if available) to fill this demand. If the warehouse does not have an uncommitted unit to fill an arriving demand, the warehouse creates a backorder and adds this to the current list of outstanding orders. When inventory becomes available, the outstanding orders are filled in the sequence in which they are created. This is the so-called *virtual allocation* rule (Axsäter 1993, Graves 1996). These committed inventory units will be shipped at the retailer's order epoch. Thus, one may view the retailers' order epochs as the shipping times planned by the warehouse. Notice that under the (S, T) policy and the virtual allocation rule, the warehouse monitors the demand continuously, whereas the material is shipped periodically and the cost is evaluated at the end of each period. We assume the virtual allocation rule because of its tractability and practicality. This allocation rule may not perform effectively when the retailers have significantly different service level requirements.

We end this section by listing the sequence of events in a period. For the warehouse, (1) orders are received from retailers; (2) an order is placed with the outside supplier if the period is an order period; (3) a shipment, if any, is received; and (4) a shipment is sent to retailer j if the period is an order period for retailer j. For retailer j, order placement occurs at the beginning of any of her order periods and customer demand arrives during the period. Costs are evaluated at the end of the period for all facilities.

3. Evaluation

Under the echelon (S, T) policy, the system forms a regenerative process with a cycle of M periods. Specifically, consider a warehouse order epoch t at which the warehouse orders up to S_0. This order will arrive at the beginning of period $t+L_0$. We assume that this is the moment that each retailer j will simultaneously place the first order up to S_j in a regenerative cycle. Thus, the regenerative cycle that we consider in the subsequent analysis is $[t, t+M)$ for the warehouse and $[t+L_0, t+L_0+M)$ for each retailer j. We call t and $t+L_0$ regenerative epochs for the warehouse and the retailers, respectively. To evaluate the average total cost per period, we only need to characterize the distribution of the inventory variables in the above regenerative cycle. The long-run average total cost per period is equal to the sum of the expected total cost incurred at the warehouse in the cycle of $[t+L_0, t+L_0+M)$ and at each retailer j in the cycle of $[t+L_{[0,j]}, t+L_{[0,j]}+M)$ divided by the cycle length M.

We describe the inventory dynamics in the considered regenerative cycle. Let r be the period index in the regenerative cycle; i.e.,

$$r = 0, 1, 2, \ldots, M-1.$$

Also, define $\lfloor a \rfloor$ as the roundoff operator, which returns the greatest integer less than or equal to a, a real number. Define

$$r_j(r) = \left\lfloor \frac{r}{T_j} \right\rfloor T_j, \quad j = 0, 1, 2, \ldots, N.$$

Thus, for $r = 0, 1, \ldots, M-1$,

$$r_j(r) \in \left\{0, T_j, 2T_j, \ldots, \left\lfloor \frac{M-1}{T_j} \right\rfloor T_j\right\}.$$

Since t and $t+L_0$ are regenerative epochs for the warehouse and the retailers, respectively, the warehouse's order periods are $t+r_0(r)$ and retailer j's order periods are $t+L_0+r_j(r)$ in a cycle of M periods. As we shall see, the order decision for the warehouse at the beginning of period $t+r_0(r)$ will directly affect the inventory amount sent to retailer j at the beginning of period $t+L_0+r_j(r)$.

We first define inventory variables for the warehouse.

$IOP_0(n)$ = echelon inventory order position at the beginning of period n,

$IN_0(n)$ = echelon inventory level at the beginning of period n,

$\mathrm{IL}_0(n)$ = echelon inventory level at the end of period n.

Let $D_0[s,t) = \sum_{j=1}^{N} D_j[s,t)$ and $D_0[s,t] = \sum_{j=1}^{N} D_j[s,t]$. The inventory dynamics for the warehouse are

$$IOP_0(t+r_0(r)) = S_0, \tag{1}$$

$$IOP_0(t+r) = IOP_0(t+r_0(r)) - D_0[t+r_0(r), t+r) = S_0 - D_0[t+r_0(r), t+r), \tag{2}$$

$$IN_0(t+L_0+r) = IOP_0(t+r) - D_0[t+r, t+L_0+r), \tag{3}$$

$$\mathrm{IL}_0(t+L_0+r) = IOP_0(t+r) - D_0[t+r, t+L_0+r]. \tag{4}$$

Equation (1) means that the warehouse's echelon inventory order position after ordering is equal to S_0. Equation (2) shows the warehouse's echelon inventory order position at any given time $t+r$ in the regenerative cycle. Equations (3) and (4) specify the echelon inventory level of the warehouse at the beginning and the end of period $t+L_0+r$, respectively.

Now we consider retailer j. We define inventory variables for retailer j.

$IOP_j(n)$ = inventory order position at the beginning of period n,

$IN_j(n)$ = inventory level at the beginning of period n,

$\mathrm{IL}_j(n)$ = inventory level at the end of period n.

Notice that under the virtual allocation rule, $IOP_j(n)$ is always equal to S_j because when a unit of demand arrives, this information is instantaneously transferred to the warehouse as if the retailer "virtually" orders whenever demand occurs. However, at retailer j's order epoch, the warehouse may not be able to fully fill retailer j's order, and thus backorders occur. We define IP_j, B_0, and B_{0j} to describe the warehouse backorders:

$\mathrm{IP}_j(n) =$ inventory in-transit position at the beginning of period n,

$B_0(n) =$ total number of warehouse backorders at the beginning of period n,

$B_{0j}(n) =$ number of warehouse backorders that belong to retailer j.

When n is the order period of retailer j, the difference between $IOP_j(n)$ and $\mathrm{IP}_j(n)$ is the warehouse's unfilled demand coming from retailer j. That is, $B_{0j}(n) = IOP_j(n) - \mathrm{IP}_j(n)$. Thus, the total number of warehouse backorders is $B_0(n) = \sum_{j=1}^N B_{0j}(n)$.

The corresponding regenerative cycle for retailer j includes periods $t + L_0 + r$, $r = 0, 1, \ldots, M-1$. Suppose that $\mathrm{IP}_j(t + L_0 + r)$ is known; it will determine $\mathrm{IL}_j(t + L_{[0,j]} + r)$ as follows:

$$\mathrm{IL}_j(t+L_{[0,j]}+r) = \mathrm{IP}_j(t+L_0+r) - D_j[t+L_0+r, t+L_{[0,j]}+r]. \quad (5)$$

After all IL_j in each period of the regenerative cycle is obtained, the total cost of the system per period is

$$C(\mathbf{S},\mathbf{T}) = \sum_{j=0}^N \frac{K_j}{T_j} + \frac{1}{M}\sum_{r=0}^{M-1} \mathsf{E}\Bigg[h_0 \mathrm{IL}_0(t+L_0+r) + \sum_{j=1}^N h_j \mathrm{IL}_j(t+L_{[0,j]}+r) + (b_j+H_j)[\mathrm{IL}_j(t+L_{[0,j]}+r)]^-\Bigg], \quad (6)$$

where $[x]^- = \max\{0, -x\}$. The first term in $C(\mathbf{S},\mathbf{T})$ represents the average total fixed cost per period[2] and the rest represents the average inventory holding and backorder cost per period.

Our remaining task is to characterize $\mathrm{IP}_j(t + L_0 + r)$. First, for retailer j, notice that $\mathrm{IP}_j(t + L_0 + r)$ depends on $\mathrm{IP}_j(t + L_0 + r_j(r))$; i.e.,

$$\mathrm{IP}_j(t+L_0+r) = \mathrm{IP}_j(t+L_0+r_j(r)) - D_j[t+L_0+r_j(r), t+L_0+r). \quad (7)$$

Since retailer j may not be able to receive the full quantity it ordered in period $t + L_0 + r_j(r)$, $\mathrm{IP}_j(t + L_0 + r_j(r))$ is not necessarily equal to S_j. More specifically,

$$\mathrm{IP}_j(t+L_0+r_j(r)) = IOP_j(t+L_0+r_j(r)) - B_{0j}(t+L_0+r_j(r)) = S_j - B_{0j}(t+L_0+r_j(r)). \quad (8)$$

To characterize $B_{0j}(t + L_0 + r_j(r))$, we first have to characterize $B_0(t + L_0 + r_j(r))$. Define $IN'_0(n)$ and $IOP'_0(n)$ as the *local* inventory level and inventory order position for the warehouse at the beginning of period n, respectively, under the virtual allocation rule. By definition,

$$\begin{aligned} B_0(t+L_0+r_j(r)) &= [IN'_0(t+L_0+r_j(r))]^- \\ &= \big[IOP'_0(t+r_j(r)) - D_0[t+r_j(r), t+L_0+r_j(r))\big]^- \\ &= \Bigg[IOP_0(t+r_j(r)) - D_0[t+r_j(r), t+L_0+r_j(r)) - \sum_{i=1}^N IOP_i(t+r_j(r))\Bigg]^- \\ &= \Bigg[IOP_0(t+r_j(r)) - D_0[t+r_j(r), t+L_0+r_j(r)) - \sum_{i=1}^N S_i\Bigg]^-. \end{aligned} \quad (9)$$

Furthermore, from (2), we can rewrite (9) as follows:

$$\begin{aligned} B_0(t+L_0+r_j(r)) &= \Bigg[S_0 - D_0[t+r_0(r_j(r)), t+r_j(r)) - D_0[t+r_j(r), t+L_0+r_j(r)) - \sum_{j=1}^N S_j\Bigg]^- \\ &= \big[s_0 - D_0[t+r_0(r_j(r)), t+r_j(r)) - D_0[t+r_j(r), t+L_0+r_j(r))\big]^- \\ &= \big[s_0 - D_0[t+r_0(r_j(r)), t+L_0+r_j(r))\big]^- \end{aligned} \quad (10)$$

where $s_0 = S_0 - \sum_{j=1}^N S_j$, the local base-stock level for the warehouse. Since $r_0(r_j(r)) \leqslant r_j(r)$, the time interval in D_0 in (10) must be at least L_0 periods. Specifically, $0 \leqslant r_j(r) - r_0(r_j(r)) < T_0$ and $r_j(r) - r_0(r_j(r)) = 0$ when T_j is an integer multiple of T_0.

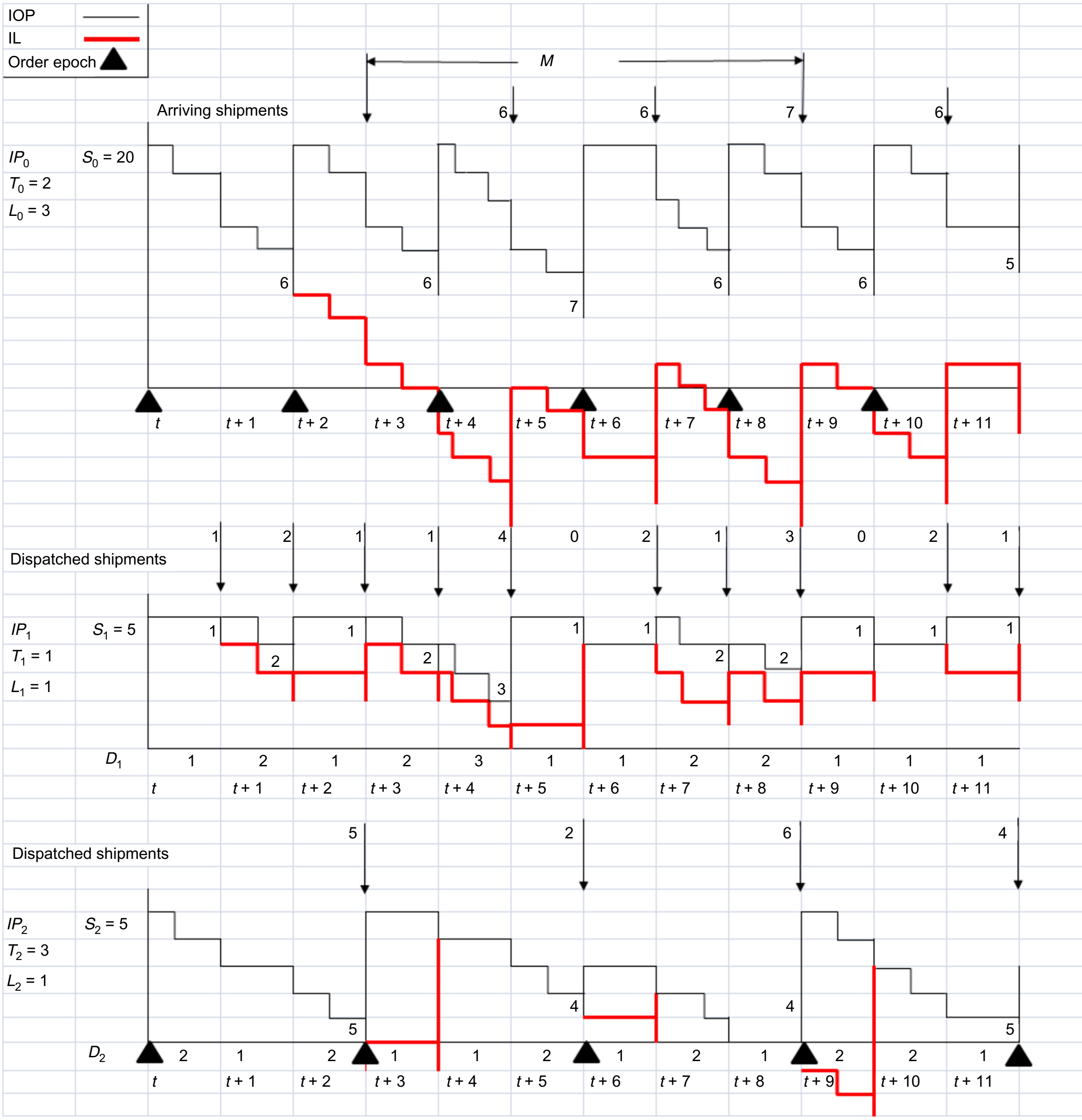
Under the virtual allocation rule, we can apply binomial disaggregation on B_0 to obtain the distribution of B_{0j} because demand arrivals at each retailer follow an independent Poisson process. That is, for any period n, we have

$$\mathsf{P}(B_{0j}(n) = k \mid B_0(n) = m) = \binom{m}{k}\left(\frac{\lambda_j}{\lambda_0}\right)^k \left(\frac{\lambda_0 - \lambda_j}{\lambda_0}\right)^{m-k}, \quad k = 0, 1, \ldots, m, \quad (11)$$

where $\lambda_0 = \sum_{j=1}^N \lambda_j$. We refer the reader to Simon (1981) for a detailed description of the binomial disaggregation technique. Since this conditional probability is independent of time, we shall omit the period index n in the further analysis. Consequently, the distribution of B_{0j} is

$$\mathsf{P}(B_{0j}(t+L_0+r_j(r)) = k) = \sum_{m=k}^{\infty} \mathsf{P}(B_0(t+L_0+r_j(r)) = m)\,\mathsf{P}(B_{0j} = k \mid B_0 = m)$$

Figure 1. (Color online) Inventory variables for a one-warehouse-two-retailer system under the (S, T) policy.



$$= \sum_{m=k}^{\infty} \mathrm{P}(D_0[L_0 + r_j(r) - r_0(r_j(r))) = s_0 + m) \binom{m}{k} \cdot \left(\frac{\lambda_j}{\lambda_0}\right)^k \left(\frac{\lambda_0 - \lambda_j}{\lambda_0}\right)^{m-k}. \quad (12)$$

Note that the distribution of B_{0j} is independent of time period t but depends on $\mathbf{S}$, T_0, and T_j. After B_{0j} is characterized, we can use (7) and (8) to further obtain the distributions of $\mathrm{IP}_j(t + L_0 + r)$ and $IP_j(t + L_0 + r_j(r))$, which also depend on $\mathbf{S}$, T_0, and T_j only. Finally, we apply (5) to obtain the distribution of IL_j for each period in the considered regenerative cycle. The average total cost per period can be calculated from (6).

Figure 1 illustrates the system dynamics and inventory variables in a two-retailer system. In this example, $(S_0, T_0) = (20, 2)$, $(S_1, T_1) = (5, 1)$, and $(S_2, T_2) = (5, 3)$; $L_0 = 3$ and $L_1 = L_2 = 1$. A regenerative cycle for the warehouse starts from period t, where the warehouse just placed an order, to period $t + 5$ and for the retailers starts from period $t + 3$ to period $t + 8$. One can verify the inventory variables through this illustrative example.

A Bottom-Up Evaluation Scheme. We provide a bottom-up recursion to simplify the evaluation of $C(\mathbf{S}, \mathbf{T})$

and to facilitate the subsequent analysis. This procedure is to first evaluate retailer j's cost by assuming that the retailer has ample supply. Next, we evaluate the echelon cost of the warehouse (which is equivalent to the total system cost) by considering the retailers' inventory positions. More specifically, let $\mathbb{M}_x(y)$ be an operator that returns the remainder of y divided by x, where x is a positive integer and y is a nonnegative integer. Thus, $\mathbb{M}_{T_j}(r)$ can be viewed as the number of periods between the order period $r_j(r)$ and the current period r for facility j.

Consider a given warehouse regenerative epoch t and retailer regenerative epoch $t+L_0$. For a given inventory position $\mathrm{IP}_j(t+L_0+r_j(r)) = y$ at the order epoch and $j=1,\ldots,N$; let $G_j(y,T_j,r)$ denote the expected inventory holding and backorder cost at retailer j in the rth period within a regenerative cycle. We have

$$\begin{aligned} G_j(y,T_j,r) &= \mathsf{E}\big[h_j(\mathrm{IL}_j(t+L_{[0,j]}+r)) \\ &\qquad + (b_j+H_j)(\mathrm{IL}_j(t+L_{[0,j]}+r))^-\big] \\ &= \mathsf{E}\big[h_j(\mathrm{IP}_j(t+L_0+r_j(r)) \\ &\qquad - D_j[t+L_0+r_j(r), t+L_{[0,j]}+r]) \\ &\qquad + (b_j+H_j)(\mathrm{IP}_j(t+L_0+r_j(r)) \\ &\qquad - D_j[t+L_0+r_j(r), t+L_{[0,j]}+r])^-\big] \\ &= \mathsf{E}\big[h_j(y-D_j[L_j+\mathbb{M}_{T_j}(r)]) \\ &\qquad + (b_j+H_j)(y-D_j[L_j+\mathbb{M}_{T_j}(r)])^-\big], \end{aligned} \tag{13}$$

where $D_j[\tau)$ and $D_j[\tau]$ denote the total demand in τ and $\tau+1$ periods, respectively.

Similarly, we define $G_0(\mathbf{S},\mathbf{T},r)$, the expected system-wide inventory cost in the rth period within a regenerative cycle. That is,

$$\begin{aligned} G_0(\mathbf{S},\mathbf{T},r) &= \mathsf{E}\Bigg[h_0(\mathrm{IL}_0(t+L_0+r)) \\ &\qquad + \sum_{j=1}^{N} G_j(\mathrm{IP}_j(t+L_0+r_j(r)),T_j,r)\Bigg] \\ &= \mathsf{E}\Bigg[h_0(\mathrm{IP}_0(r_0(r)) - D_0[t+r_0(r),t+L_0+r]) \\ &\qquad + \sum_{j=1}^{N} G_j(\mathrm{IP}_j(t+L_0+r_j(r)),T_j,r)\Bigg] \\ &= \mathsf{E}\Bigg[h_0(S_0 - D_0[L_0+\mathbb{M}_{T_0}(r)]) \\ &\qquad + \sum_{j=1}^{N} G_j(S_j - B_{0j}(L_0+r_j(r)),T_j,r)\Bigg], \end{aligned} \tag{14}$$

where $B_{0j}(L_0+r_j(r))$ can be found from (12).

Let

$$G(\mathbf{S},\mathbf{T}) \stackrel{\text{def}}{=} \frac{1}{M}\sum_{r=0}^{M-1} G_0(\mathbf{S},\mathbf{T},r).$$

PROPOSITION 1. *For given echelon* (S,T) *policies with parameters* $(\mathbf{S},\mathbf{T})$*, the average total cost per period is*

$$C(\mathbf{S},\mathbf{T}) = \sum_{j=0}^{N} \frac{K_j}{T_j} + G(\mathbf{S},\mathbf{T}),$$

where $\sum_{j=0}^{N}(K_j/T_j)$ *is the average total fixed order cost per period and* $G(\mathbf{S},\mathbf{T})$ *is the average inventory holding and backorder cost per period.*

Proposition 1 can be easily verified because the total cost obtained from Equations (13) and (14) is equivalent to the cost in (6).

4. Optimization

This section discusses how to obtain the optimal (S,T) policy. When the reorder intervals are fixed, we show that Axsäter's (1990) algorithm can be revised to obtain the optimal echelon base-stock levels (see Appendix A). Our focus here is to show how to obtain the optimal reorder intervals. Our approach is to derive bounds for the optimal parameters and enumerate solutions within the bounds. In §4.1, we allocate the total system cost to each facility. In §4.2, we construct a lower bound function to the allocated facility cost. In §4.3, we derive the parameter bounds by using these lower bound functions.

4.1. Decomposition of the System Cost

This section shows how to allocate the total system cost to each facility. Our starting point is the parameter allocation scheme suggested by Chen and Zheng (1994), who studied the distribution system with the review period equal to one for all facilities. Since our model is more general, we have to extend their approach to construct the lower bound.

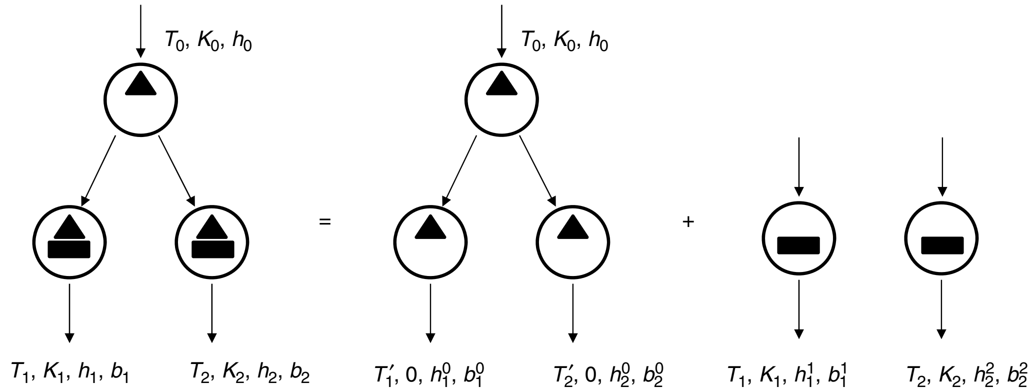
Following Chen and Zheng, imagine that the final product carried at retailer j, $j=1,\ldots,N$ is composed of two components, 0 and j. The warehouse delivers component 0 to retailer j, where component j is added to component 0 to produce the final product. Figure 2 illustrates this idea. Here, the triangle represents component 0 and the rectangle represents component j. We allocate the holding cost and backorder cost of the final product to each component. More specifically, we use a superscript to represent the component index and a subscript to represent the final product index. For $j=1,\ldots,N$,

$$h_j^0 + h_j^j = h_j, \quad \text{and} \quad b_j^0 + b_j^j = b_j.$$

To facilitate the subsequent analysis, we define the cost functions of components 0 and j at retailer j as follows: For a given $\mathrm{IP}_j(t+L_0+r_j(r)) = y$,

$$\begin{aligned} g_j^0(y,T_j,r) &= \mathsf{E}\big[h_j^0(y-D_j[L_j+\mathbb{M}_{T_j}(r)]) \\ &\qquad + (b_j^0+h_0+h_j^0)(y-D_j[L_j+\mathbb{M}_{T_j}(r)])^-\big], \\ g_j^j(y,T_j,r) &= \mathsf{E}\big[h_j^j(y-D_j[L_j+\mathbb{M}_{T_j}(r)]) \\ &\qquad + (b_j^j+h_j^j)(y-D_j[L_j+\mathbb{M}_{T_j}(r)])^-\big]. \end{aligned}$$

Figure 2. Parameter allocation for the system.



Clearly,

$$g_j^0(y, T_j, r) + g_j^j(y, T_j, r) = G_j(y, T_j, r).$$

Thus, the expected inventory holding and backorder cost per period for any given set of the (S, T) policies can be expressed as follows:

$$G(\mathbf{S},\mathbf{T}) = \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}\left[\left(h_0(\mathrm{IL}_0(t+L_0+r)) + \sum_{j=1}^{N}(g_j^0(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r) + g_j^j(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r))\right)\right]\right\}. \quad (15)$$

The first two terms on the right-hand side of Equation (15) are the inventory holding and backorder cost for component 0 in the distribution system. The last term g_j^j function is the inventory holding and backorder cost for component j in the single-stage system, $j = 1, \ldots, N$.

In Chen and Zheng (1994), the authors can solve $N+1$ separable systems (one distribution system and N single-stage systems) after the above cost allocation scheme because the reorder intervals (review periods) are implicitly assumed to be one. However, our problem is more complicated because for each fixed r in the regenerative cycle, the cost function of component 0 is a function of $\mathbf{T}$, and the cost function of component j is a function of T_0 and T_j. Thus, we cannot fully decouple the system as Chen and Zheng did.

Below we derive a new result that further decomposes the $g_j^0(\cdot, T_j, r)$ function, i.e., the cost of component 0 incurred at retailer j. Notice that

$$g_j^0(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r) = g_j^0(\mathrm{IP}_j(t+L_0+r), T_j, 0).$$

Thus, Equation (15) can be rewritten as

$$G(\mathbf{S},\mathbf{T}) = \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}\left[h_0(\mathrm{IL}_0(t+L_0+r)) + \sum_{j=1}^{N}(g_j^0(\mathrm{IP}_j(t+L_0+r), T_j, 0) + g_j^j(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r))\right]\right\}. \quad (16)$$

From (7) and (8), recall that $\mathrm{IP}_j(t+L_0+r)$ is a function of $\mathbf{S}$, T_0, and T_j. Let us focus on the $g_j^0(\cdot, T_j, 0)$ function. For notational simplicity, without confusion, we omit the last two arguments in the g_j^0 function; i.e., $g_j^0(\cdot, T_j, 0) = g_j^0(\cdot)$. We define the following functions to decouple the $g_j^0(\cdot)$ function.

$$g_j^0(y) = \mathsf{E}\left[h_j^0(y - D[L_j]) + (b_j^0 + h_0 + h_j^0)(y - D[L_j])^-\right],$$

$$S_j = \arg\min_y g_j^0(y).$$

Let

$$g_{jj}^0(y) = \begin{cases} g_j^0(S_j), & \text{if } y \leqslant S_j, \\ g_j^0(y), & \text{otherwise,} \end{cases} \quad (17)$$

and

$$g_{j0}^0(y) = g_j^0(y) - g_{jj}^0(y).$$

Thus, $g_j^0(y) = g_{j0}^0(y) + g_{jj}^0(y)$ for all y. Note that $g_{jj}^0(y)$ and $g_{j0}^0(y)$ are both convex functions: $g_{jj}^0(y)$ is constant for $y \leqslant S_j$ and convex increasing for $y > S_j$, whereas $g_{j0}^0(y)$ is convex decreasing for $y \leqslant S_j$ and zero for $y > S_j$.

With this decomposition scheme, Equation (16) can be expressed as

$$G(\mathbf{S},\mathbf{T}) = \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}\left[h_0(\mathrm{IL}_0(t+L_0+r)) + \sum_{j=1}^{N}\left(g_{j0}^0(\mathrm{IP}_j(t+L_0+r)) + g_{jj}^0(\mathrm{IP}_j(t+L_0+r)) + g_j^j(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r)\right)\right]\right\} \quad (18)$$

$$= g_0(\mathbf{S},\mathbf{T}) + \sum_{j=1}^{N} g_j(\mathbf{S},\mathbf{T}), \quad (19)$$

where

$$g_0(\mathbf{S},\mathbf{T}) = \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}\left[h_0(\mathrm{IL}_0(t+L_0+r)) + \sum_{j=1}^{N}(g_{j0}^0(\mathrm{IP}_j(t+L_0+r)))\right]\right\}, \quad (20)$$

$$g_j(\mathbf{S},\mathbf{T}) = \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}\left[g_{jj}^0(\mathrm{IP}_j(t+L_0+r)) + g_j^j(\mathrm{IP}_j(t+L_0+r_j(r)), T_j, r)\right]\right\}. \quad (21)$$

This completes our decomposition scheme. Here, $g_0(\mathbf{S},\mathbf{T})$ is the allocated cost for the warehouse and $g_j(\mathbf{S},\mathbf{T})$ is the allocated cost for retailer j.

4.2. Lower Bound for the Allocated Facility Cost

This section develops a lower bound to the allocated cost function $g_j(\mathbf{S},\mathbf{T})$, $j=0,1,\ldots,N$. We first consider the allocated warehouse cost.

$$\begin{aligned} g_0(\mathbf{S},\mathbf{T}) &= \frac{1}{M}\sum_{r=0}^{M-1} \mathsf{E}\left[h_0\, \mathrm{IL}_0(t+L_0+r) + \sum_{j=1}^{N} (g_{j0}^0(\mathrm{IP}_j(t+L_0+r)))\right] \\ &\geqslant \frac{1}{M}\sum_{r=0}^{M-1} \mathsf{E}\left[h_0\, \mathrm{IL}_0(t+L_0+r) + \min_{\sum_j y_j(r) \leqslant \sum_j \mathrm{IP}_j(t+L_0+r)} \sum_{j=1}^{N} g_{j0}^0(y_j(r))\right] \\ &\geqslant \frac{1}{M}\sum_{r=0}^{M-1} \mathsf{E}\left[h_0\, \mathrm{IL}_0(t+L_0+r) + \min_{\sum_j y_j(r) \leqslant IN_0(t+L_0+r)} \sum_{j=1}^{N} g_{j0}^0(y_j(r))\right] \\ &= \frac{1}{T_0}\sum_{r=0}^{T_0-1} \mathsf{E}\left[h_0\, \mathrm{IL}_0(t+L_0+r) + \min_{\sum_j y_j(r) \leqslant IN_0(t+L_0+r)} \sum_{j=1}^{N} g_{j0}^0(y_j(r))\right] \\ &\equiv \underline{g}_0(S_0, T_0). \end{aligned}$$

The first inequality holds because we reoptimize the inventory allocation between the retailers. (This corresponds to the so-called balance assumption used to develop a lower bound to the system cost in Clark and Scarf 1960.) The second inequality holds because

$$IN_0(t+L_0+r) \geqslant \sum_j \mathrm{IP}_j(t+L_0+r).$$

The second equality holds because $\mathrm{IL}_0(t+L_0+r)$ and $IN_0(t+L_0+r)$ are cyclic with the cycle length of T_0 periods.

The following convexity result leads to the lower bound function.

Proposition 2. *For fixed T_0, $\underline{g}_0(S_0,T_0)$ is convex in S_0.*

Let $S_0(T_0) = \arg\min_{S_0} \underline{g}_0(S_0,T_0)$, and $\underline{g}_0(T_0) = \underline{g}_0(S_0(T_0),T_0)$. We have $g_0(\mathbf{S},\mathbf{T}) \geqslant \underline{g}_0(T_0)$. The function $\underline{g}_0(T_0)$ is the lower bound to the allocated warehouse cost.

We next develop a lower bound for the allocated retailer cost.

Proposition 3. *For a given T_j, $\sum_{r=0}^{T_j-1} g_j^j(y,T_j,r)$ is convex in y.*

Let $S_j(T_j) = \arg\min_y (1/T_j)\sum_{r=0}^{T_j-1} g_j^j(y,T_j,r)$. We have

$$\begin{aligned} g_j(\mathbf{S},\mathbf{T}) &= \frac{1}{M}\left\{\sum_{r=0}^{M-1} \mathsf{E}[g_{jj}^0(\mathrm{IP}_j(t+L_0+r)) + g_j^j(\mathrm{IP}_j(t+L_0+r_j(r)),T_j,r)]\right\} \\ &\geqslant \frac{1}{T_j}\left\{\sum_{r=0}^{T_j-1} \mathsf{E}[g_{jj}^0(S_j) + g_j^j(S_j(T_j),T_j,r)]\right\} \\ &\equiv \underline{g}_j(T_j). \end{aligned}$$

The inequality holds because the first term $g_{jj}^0(S_j)$ is a constant, which is the smallest component 0 cost incurred at retailer j, derived in (17). The function $\underline{g}_j(T_j)$ is the lower bound to the allocated retailer cost.

Define $\underline{c}_j(T_j) = (K_j/T_j) + \underline{g}_j(T_j)$ for $j=0,1,\ldots,N$. We summarize the lower bound result.

Theorem 4. $C(\mathbf{S},\mathbf{T}) \geqslant \sum_{j=0}^{N} \underline{c}_j(T_j)$.

Note that the lower bound to the system cost is a sum of $N+1$ separable functions. This result leads to the optimization procedure discussed in the next section.

4.3. Bounds on the Optimal Reorder Intervals

We can use Theorem 4 to derive the bounds for T_j^*, $j=0,1,\ldots,N$. Suppose that we can find the minimizer of $\underline{c}_j(T_j)$ and let the corresponding minimum cost be $\underline{c}_j$, for $j=0,1,\ldots,N$. Denote C^h as the cost obtained from any heuristic policy. (In §5.2, we will suggest a heuristic.) Then

$$C^h \geqslant C(\mathbf{S}^*,\mathbf{T}^*) \geqslant \sum_{j=0}^{N} \underline{c}_j(T_j^*) \geqslant \underline{c}_j(T_j^*) + \sum_{i\neq j} \underline{c}_i.$$

An upper bound $\bar{T}_j$ and a lower bound $\underline{T}_j$ to T_j^* can be obtained by solving the following inequalities:

$$\begin{aligned} \bar{T}_j &= \max\left\{T_j \;\middle|\; \underline{c}_j(T_j) \leqslant C^h - \sum_{i\neq j}\underline{c}_i\right\}, \\ \underline{T}_j &= \min\left\{T_j \;\middle|\; \underline{c}_j(T_j) \leqslant C^h - \sum_{i\neq j}\underline{c}_i\right\}. \end{aligned} \quad (22)$$

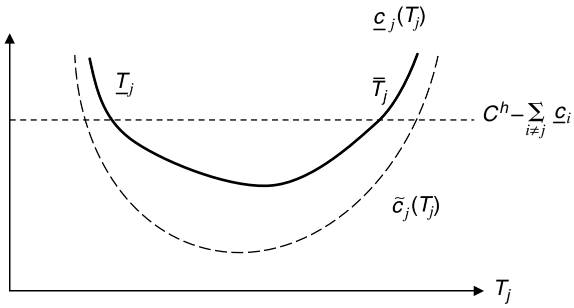
The logic of constructing these bounds is illustrated in Figure 3. $\bar{T}_j$ and $\underline{T}_j$ are the two points of intersection of the constant function $(C^h - \sum_{i\neq j}\underline{c}_i)$ and the $\underline{c}_j(T_j)$ function.

The challenge of using this approach is that, unfortunately, $\underline{c}_j(T_j)$ is not quasiconvex in T_j (Liu and Song 2012). Thus, it is very difficult to obtain the minimizer and the minimum value $\underline{c}_j$. To overcome this difficulty, we construct a quasiconvex function that bounds the $\underline{c}_j(T_j)$ function, which can be used to find $\bar{T}_j$ and $\underline{T}_j$.

Define

$$\tilde{c}_0(T_0) = \frac{K_0}{T_0} + \min_y\left\{\frac{1}{T_0}\sum_{r=0}^{T_0-1}\left[h_0(y-(L_0+r+1)\lambda_0) + \min_{\sum_j y_j(r)\leqslant y-(L_0+r)\lambda_0} \sum_{j=1}^{N} g_{j0}^0(y_j(r))\right]\right\}.$$

Figure 3. The $\underline{c}_j(T_j)$ and $\tilde{c}_j(T_j)$ functions.



For $j = 1, \ldots, N$,

$$\tilde{c}_j(T_j) = \frac{K_j}{T_j} + \min_y \left\{ \frac{1}{T_j} \sum_{r=0}^{T_j - 1} h_j^j (y - \lambda_j(L_j + r + 1)) + (h_j^j + b_j^j)(y - \lambda_j(L_j + r + 1))^- \right\} + g_{jj}^0(S_j).$$

Proposition 5. *For $j = 0, 1, \ldots, N$, (1) $\underline{c}_j(T_j) \geqslant \tilde{c}_j(T_j)$ for all T_j; (2) $\tilde{c}_j(T_j)$ is quasiconvex in T_j.*

Proposition 5(1) states that the lower bound function $\underline{c}_j(T_j)$ is bounded below by the function $\tilde{c}_j(T_j)$, which is obtained from Jensen's inequality. Proposition 5(2) shows that this lower bound function $\tilde{c}_j(T_j)$ is quasiconvex in T_j. Figure 3 illustrates these functions.

We can use $\tilde{c}_j(T_j)$ to search for the minimum value of $\underline{c}_j(T_j)$, or $\underline{c}_j$. Specifically, we evaluate both $\underline{c}_j(T_j)$ and $\tilde{c}_j(T_j)$ sequentially from $T_j = 1, 2, \ldots$ until T_j reaches a value, say, s, such that $\tilde{c}_j(s) \geqslant \min_{t \in \{1,2,\ldots,s\}} \underline{c}_j(t)$. From Proposition 5(1), we have $\underline{c}_j(s) \geqslant \tilde{c}_j(s) \geqslant \min_{t \in \{1,2,\ldots,s\}} \underline{c}_j(t)$, which implies that we cannot find a reorder interval greater than s such that the resulting cost is smaller than $\min_{t \in \{1,2,\ldots,s\}} \underline{c}_j(t)$. Thus, the minimum value will be $\underline{c}_j = \min_{t \in \{1,2,\ldots,s\}} \underline{c}_j(t)$.

After we obtain all $\underline{c}_j$, we can use $\tilde{c}_j(T_j)$ to find the parameter bounds $\underline{T}_j$ and $\bar{T}_j$ via (22). We start with an empty set $\mathcal{S}$. We evaluate both $\underline{c}_j(T_j)$ and $\tilde{c}_j(T_j)$ sequentially from $T_j = 1, 2, \ldots$, etc. If $\underline{c}_j(T_j) \leqslant C^h - \sum_{i \neq j} \underline{c}_i$, we add this T_j to the set $\mathcal{S}$. Continue this procedure until for some $T_j = s$ both conditions $\tilde{c}_j(s) \geqslant \tilde{c}_j(s-1)$ and $\tilde{c}_j(s) > C^h - \sum_{i \neq j} \underline{c}_i$ are satisfied. In this case, we do not need to consider any $T_j > s$ because $\underline{c}_j(T_j) \geqslant \tilde{c}_j(T_j) \geqslant C^h - \sum_{i \neq j} \underline{c}_i$. Thus, $\underline{T}_j$ ($\bar{T}_j$) is the minimum (maximum) value in $\mathcal{S}$.

Notice that $\underline{c}_j(T_j)$ is constructed from a specific pair of (h_j^0, h_j^j) and (b_j^0, b_j^j) with $h_j^0 + h_j^j = h_j$ and $b_j^0 + b_j^j = b_j$. In other words, by splitting h_j and b_j into different combinations, we can generate different parameter bounds. Our final upper bound $\bar{T}_j$ (lower bound $\underline{T}_j$) is the minimum (maximum) value of all upper (lower) bounds generated from these different combinations. In §5, we shall report our choices to split the h_j and b_j parameters.

Clearly, to find the optimal reorder intervals, we only need to enumerate the policies with $T_j \in [\underline{T}_j, \bar{T}_j]$ and their corresponding optimal base-stock levels. Below we prove a property that may reduce the number of feasible solutions.

Proposition 6. *If T_j^*/T_i^* or $T_i^*/T_j^* \in \mathbb{N}$ for any $i, j \in \{0, 1, \ldots, N\}$, then $T_0^* \geqslant \min\{T_1^*, \ldots, T_N^*\}$.*

Proposition 6 states that if the optimal reorder intervals form integer-ratio relationships, the reorder interval of the warehouse must be no less than the minimum of those for the retailers. This is intuitive because if the above result does not hold, the warehouse will carry inventory that cannot immediately be shipped to any retailer, causing unnecessary inventory holding costs. Thus, the system cost (both holding and fixed order costs) can always be improved by increasing T_0. We notice that this property corresponds to the "last-minute" property, i.e., the warehouse orders only when at least one retailer orders, established by Schwarz (1973) for the deterministic model.

5. Numerical Study and Heuristic

In §5.1, we examine the optimal reorder intervals and the POT policy generated from the corresponding deterministic model (referred to as the deterministic POT policy hereafter). We aim to observe properties of the optimal solution and identify conditions under which the deterministic POT policy performs less effectively. These observations are useful to suggest an effective heuristic for the stochastic OWMR model, which is presented in §5.2. In §5.3, we report on the performance of the suggested heuristic. In §5.4, we report on the quality of the parameter bounds under the proposed heuristic. A complete description of optimal solutions as well as the parameter bounds is available in Appendix C in the online companion (available as supplemental material at http://dx.doi.org/10.1287/opre.2015.1347).

5.1. The Optimal and POT Policies

Since there are many parameters in the system, we first conduct a pretest to exclude the parameters that do not have a direct impact on the reorder intervals. This pretest includes 128 two-retailer instances.[3] Our finding indicates that the optimal reorder intervals are insensitive to the change of lead times. This is intuitive because the lead time should have a direct impact on the optimal base-stock level, which is observed in this pretest. Thus, we fix lead times equal to one for the subsequent studies.

The test bed includes 432 one-warehouse-two-retailer systems, which are generated from the following parameter sets:

$$L_0 = L_1 = L_2 = 1, \quad K_0, K_1, K_2 \in \{2, 8, 32\},$$
$$h_1 = 1, h_0, h_2 \in \{0.5, 2\}, \quad b_1 = 25, b_2 \in \{25, 50\},$$
$$\lambda_1 = 3, \lambda_2 \in \{3, 6\}.$$

In these examples, we search for the optimal reorder intervals as well as the deterministic POT policies.

We use the Poisson demand rate as the demand rate for the deterministic model and apply Roundy's (1985) algorithm to generate POT reorder intervals, denoted as $\mathbf{T}^d = (T_0^d, T_1^d, \ldots, T_N^d)$. We use the algorithm shown in Appendix A to find the corresponding optimal base-stock levels. Let the resulting cost be $C(\mathbf{T}^d)$. To find the optimal (S, T) policy, we use the procedure in §4 to find the bounds for the reorder intervals. (The heuristic used in this procedure will be introduced in §5.2.) For each solution of the reorder intervals within the bounds, we find the corresponding optimal base-stock levels and evaluate the total cost. Let $(\mathbf{S}^*, \mathbf{T}^*)$ and $C(\mathbf{S}^*, \mathbf{T}^*)$ denote the optimal solution and cost, respectively. Define the optimality gap as

$$\frac{C(\mathbf{T}^d) - C(\mathbf{S}^*, \mathbf{T}^*)}{C(\mathbf{S}^*, \mathbf{T}^*)} \times 100\%.$$

The average optimality gap is 5.70% with a maximum gap of 36.63%. Although the overall performance is reasonably effective, to our surprise, the POT solution could be significantly suboptimal. More specifically, there are 90 instances (20.8% of the 432 instances) whose optimality gaps are more than 10%. Below we provide several observations on the optimal reorder intervals and the POT solution.

(1) We find that in most scenarios the optimal policy satisfies the integer-ratio property, but there are exceptions. For example, when $(K_0, K_1, K_2) = (8, 32, 8)$, $(h_0, h_1, h_2) = (0.5, 1, 2)$, $(b_1, b_2) = (25, 50)$, and $(\lambda_1, \lambda_2) = (3, 6)$, the optimal reorder intervals are $(T_0^*, T_1^*, T_2^*) = (2, 3, 1)$. Nevertheless, an integer-ratio policy is still a good candidate when designing a heuristic for the reorder intervals. Among the 432 instances we tested, there are only 7 instances whose optimal solutions are not an integer-ratio policy.

(2) We find that the cost ratio $K_j/(h_j\lambda_j)$ is a key factor that affects the effectiveness of the deterministic POT policy. More specifically, when $K_0/(h_0\lambda_0)$ is significantly smaller than one of the cost ratios $K_j/(h_j\lambda_j)$, the deterministic POT policy tends to perform worse. For example, for the 98 instances with

$$\frac{\max\{K_1/(h_1\lambda_1), K_2/(h_2\lambda_2)\}}{K_0/(h_0\lambda_0)} > 20,$$

the average optimality gap is 10.59% with a maximum gap of 36.63%.

When $K_0/(h_0\lambda_0) < K_j/(h_j\lambda_j)$ for any $j \in \{1, 2\}$, it is likely that $T_0^d = T_1^d < T_2^d$ or $T_0^d = T_2^d < T_1^d$. However, we observe that in the optimal solution T_0^*, T_1^*, and T_2^* tend to be the same. One reason that leads to this difference is the order (demand) pooling effect. In the deterministic model, since there is no demand variability, the retailers are clustered only based on the cost ratios. However, with stochastic demand, the warehouse can choose a larger reorder interval to consolidate the retailers' orders to take advantage of demand pooling. Thus, the optimal reorder intervals tend to be the same in the stochastic demand model even when $K_j/(h_j\lambda_j)$ are quite different among the retailers.

(3) It is conceivable that when the backorder cost b_j increases, retailer j's optimal reorder interval T_j^* will decrease. This is because a shorter reorder interval makes the retailer more responsive to the demand, which will reduce the backorders. Interestingly, we find that a larger backorder cost at one retailer may also shorten the optimal reorder intervals of the other retailers. For example, in the case with $(K_0, K_1, K_2) = (8, 8, 32)$, $(h_0, h_1, h_2) = (0.5, 1, 0.5)$, and $b_1 = 25$, when b_2 increases from 25 to 50, the optimal reorder intervals are changed from $(T_0^*, T_1^*, T_2^*) = (4, 4, 4)$ to $(3, 3, 3)$. This phenomenon is due to the benefit of order coordination. From the system's perspective, although reducing T_1^* from 4 to 3 may increase the fixed ordering cost, this additional cost is offset by the benefit received from order coordination between these two retailers. We find that the demand rate has a similar effect: When the demand rate at a retailer increases, the optimal reorder intervals of all retailers become smaller.

(4) Overall, compared with the optimal base-stock levels, the optimal reorder intervals are relatively insensitive to changes in the system parameters. In other words, if a supply chain manager chooses an effective set of reorder intervals, the system can achieve a high efficiency by adjusting the base-stock levels. From a management perspective, the supply chain manager may view the reorder interval decision as a medium-term tactical decision, whereas the inventory decision as a short-term operational planning that can be changed more frequently according to the system states.

5.2. The Heuristic for the Stochastic OWMR Model

Based on the above observations, we conclude that an integer-ratio policy is a good candidate for a heuristic policy and that the POT clustering step based on the ratio $K_j/(h_j\lambda_j)$ is an effective method for clustering facilities, except when $K_0/(h_0\lambda_0) < K_j/(h_j\lambda_j)$. In addition, we should also utilize the lower bound cost function to incorporate the stochastic demand effect.

We suggest the following two-step algorithm to generate a heuristic integer-ratio policy. The first step is to cluster facilities according to two clustering schemes. The first scheme is to follow Roundy's (1985) algorithm where we cluster the retailers into one of three sets, G, E, and L. The set G (E, L, respectively) contains the facilities whose reorder intervals are larger than (equal to, less than, respectively) those of the warehouse. The warehouse by default is assigned to the set E. The second scheme is to simply cluster all facilities into the set E (that is, all facilities use the same reorder interval). This takes into account the benefit of pooling when all facilities use the same reorder interval. Thus, after the first step, we obtain two clustering outputs.

The second step is to generate an integer-ratio policy for each clustering output by solving a series of single-stage problems. More specifically, we solve the problems $\arg\min_T\{\sum_{j\in E} \underline{c}_j(T)\}$ for facility $j \in E$ and $\arg\min_{T_j} \underline{c}_j(T_j)$ for facility $j \notin E$.[4] In other words, let i

represent the number of problems we need to solve in the second step and $|E|$ denote the number of facilities in the set E. Then $i = (N+1) - (|E|) + 1 = N - |E| + 2$. After we solve these i problems, we sort the resulting solutions from the smallest to the largest: $T_{(1)} \leqslant T_{(2)} \leqslant \cdots \leqslant T_{(i)}$. Here, (m) represents the set that includes facility j with the solution $T_{(m)}$, $m = 1, 2, \ldots, i$. Now we generate the integer-ratio solution as follows: let $\tilde{T}_{(1)} = T_{(1)}$. For $m = 2, \ldots, i$, we set

$$\tilde{T}_{(m)} = \begin{cases} \arg\min\limits_{T = q\tilde{T}_{(m-1)}} \left\{ \sum\limits_{j \in (m)} \underline{c}_j(T) \right\}, & \text{for } (m) = E, \\ \arg\min\limits_{T = q\tilde{T}_{(m-1)}} \{\underline{c}_j(T)\}, & \text{for } j \in (m) \neq E, \end{cases}$$

where q is a positive integer such that $q\tilde{T}_{(m-1)}$ is the first minimizer for the considered cost function. Then we can get an integer-ratio solution $\tilde{T}_j = \tilde{T}_{(m)}$ for $j \in (m)$, $m = 1, \ldots, i$. Since we have two clustering results, our final heuristic solution will be the one with a smaller cost.

For the purpose of easy synchronization of shipments, supply chain firms may want to adopt POT reorder intervals (see discussions in Maxwell and Muckstadt 1985). Our heuristic approach can generate POT reorder intervals in step 2. That is, after we obtain $T_{(m)}$, we can turn these values into POT integers according to Roundy (1985). In the numerical study below, we report the performance of both heuristic policies.

5.3. Effectiveness of the Heuristic

We test the effectiveness of the heuristic solution $\tilde{\mathbf{T}} = (\tilde{T}_0, \ldots, \tilde{T}_N)$ and its corresponding best base-stock levels $\tilde{\mathbf{S}} = (\tilde{S}_0, \ldots, \tilde{S}_N)$ for the above 432 two-retailer systems. We split h_j and b_j as follows $h_j^0 = 0, h_j^j = h_j, b_j^0 = 0.5b_j, b_j^j = 0.5b_j$ to generate the lower bound functions $\underline{c}_j(T_j)$, $j = 0, \ldots, N$. We define the percentage optimality gap of the heuristic as

$$\frac{C(\tilde{\mathbf{S}}, \tilde{\mathbf{T}}) - C(\mathbf{S}^*, \mathbf{T}^*)}{C(\mathbf{S}^*, \mathbf{T}^*)} \times 100\%.$$

The average percentage optimality gap of our heuristic is 0.34% with a maximum gap of 5.95%. The heuristic solution significantly outperforms the deterministic POT policy. Remarkably, 88% of the instances have a percentage gap of less than 1%. Figure 4 shows the distribution of the percentage optimality gaps. We observe that the heuristic performs less effectively when $K_0/(\lambda_0 h_0)$ is close to one of $K_j/(\lambda_j h_j)$, $j = 1, 2$, and is significantly smaller than the other one. For example, the instance with the largest percentage gap has the following parameters: $K_0 = 2$, $K_1 = 32$, $K_2 = 2$, $h_0 = 0.5$, $h_1 = 1$, $h_2 = 0.5$, $b_1 = 25$, $b_2 = 25$, $\lambda_1 = 3$, $\lambda_2 = 6$. In this case, $K_0/(\lambda_0 h_0) = 0.44$, $K_1/(\lambda_1 h_1) = 10.67$, and $K_2/(\lambda_2 h_2) = 0.67$. The optimal solution is $(T_0^*, T_1^*, T_2^*) = (1, 3, 1)$ and the heuristic solution is $(\tilde{T}_0, \tilde{T}_1, \tilde{T}_2) = (1, 5, 1)$. Our heuristic clusters the warehouse and retailer 2 into the set E, leaving retailer 1 to the set G. This clustering result is the same as that of the optimal solution. However, our heuristic solves stage 1 as a single-stage system, whereas the optimal solution reflects the benefit of pooling of two retailers. This explains why the resulting $\tilde{T}_1$ is larger than T_1^*.

To further investigate the robustness of the heuristic, we test larger systems with $N = 4$ and $N = 8$. For $N = 4$, we choose the parameters from the two sets of parameters listed in Table 1. For the first set of parameters, we consider a scenario in which one retailer is different from the other three identical retailers; for the second set, we consider four heterogeneous retailers. The total number of instances is 128. The average (maximum) percentage optimality gap of the heuristic compared to the exact optimal (S, T) policy is 0.47% (4.25%). We do not find a significant difference between these two sets in terms of the heuristic performance.

With the same logic, we also consider two sets of parameters for $N = 8$. The first set assumes that retailers 5 to 8 have exactly the same parameters as retailer 2 in Set I with $N = 4$; the second set considers four heterogeneous types of retailers. That is, retailer i and retailer $i+4$ have the same parameters as retailer i in Set II with $N = 4$, $i = 1, \ldots, 4$. Again, the total number of tested instances for the eight-retailer system is 128. The average (maximum) percentage optimality for the heuristic is 0.36% (4.92%). Figure 4 shows the distribution of percentage gaps for our heuristic. It is clear that the performance of the proposed heuristic does not deteriorate when N increases.

As stated, our heuristic approach can generate POT reorder intervals different from those obtained from Roundy's algorithm. We test the effectiveness of the POT policy generated by our heuristic. For the same parameter sets tested in the numerical study, the average (maximum) POT heuristic optimality gaps for $N = 2, 4$, and 8 are 1.37% (8.61%), 0.96% (9.02%), and 0.70% (5.76%), respectively. Although the heuristic POT policies have

Table 1. System parameters for four-retailer systems.

Set I	Set II
$L_0 = L_1 = L_2 = L_3 = L_4 = 1$	$L_0 = L_1 = L_2 = L_3 = L_4 = 1$
$K_0 = 32,\ K_1 \in \{2, 8\},\ K_2 = K_3 = K_4 \in \{2, 8\}$	$K_0 = 32,\ K_1 = 2,\ K_2 = 4,\ K_3 = 8,\ K_4 = 16$
$h_1 = 1,\ h_0 \in \{0.5, 2\},\ h_2 = h_3 = h_4 \in \{0.5, 2\}$	$h_1 = 1,\ h_0 \in \{0.5, 2\},\ h_2 = h_3 = h_4 \in \{0.5, 2\}$
$b_1 = 25,\ b_2 = b_3 = b_4 \in \{25, 50\}$	$b_1 = 25,\ b_2 = b_3 = b_4 \in \{25, 50\}$
$\lambda_1 = 3,\ \lambda_2 = \lambda_3 = \lambda_4 \in \{3, 6\}$	$\lambda_1 = 3,\ \lambda_2 = \lambda_3 = \lambda_4 \in \{3, 6\}$

Figure 4. The distribution of percentage optimality gaps for the heuristic integer-ratio policies.

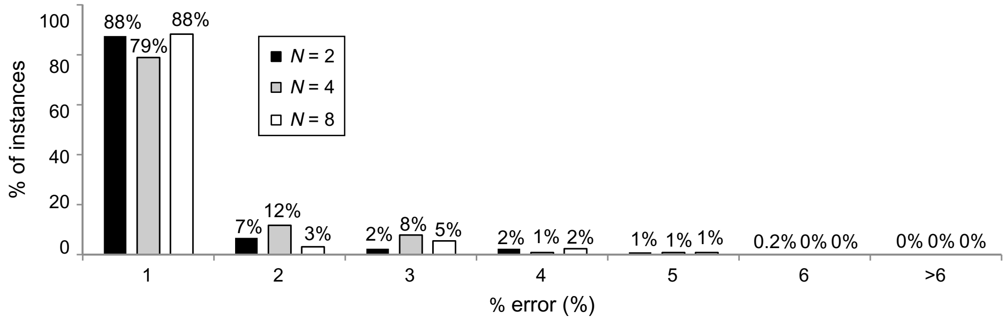
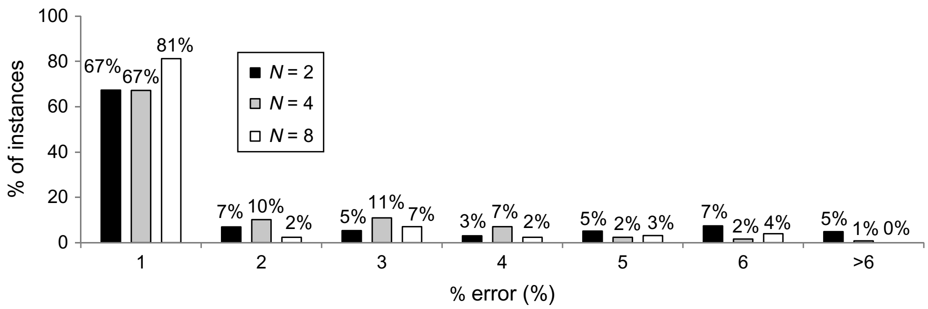


Figure 5. The distribution of percentage optimality gaps for the heuristic power-of-two policies.



larger average and maximum percentage gaps than the heuristic integer-ratio policies, the overall performance of the heuristic POT policies still is quite satisfactory. Figure 5 shows the distribution of percentage optimality gaps for the heuristic POT policies.

Our heuristic can generate an integer-ratio policy very efficiently: The average running time for one instance of two-, four-, and eight-retailer systems is 13.6 seconds, 25.9 seconds, and 98.3 seconds respectively, with a 2.3 GHz CPU. The running time includes finding the heuristic reorder intervals and the corresponding optimal base stock levels as well as evaluating the heuristic cost. This is significantly shorter than the average running time for solving the exact optimal solution.[5]

5.4. Effectiveness of the Optimal Parameter Bounds

This section examines the quality of the parameter bounds $\bar{T}_j$ and $\underline{T}_j$ obtained in §4. We first focus on the 432 two-retailer instances. We split h_j and b_j with $h_j^0 = 0$, $h_j^j = h_j$ and $b_j^0 = \alpha b_j$, $b_j^j = (1-\alpha)b_j$, where $\alpha \in \{0.2, 0.4, 0.6, 0.8\}$. For each α, we can get an upper bound and a lower bound for the optimal $\mathbf{T}^*$. Then we choose the maximum of the lower bounds and the minimum of the upper bounds as the final parameter bounds.

We denote $\Delta_j = \bar{T}_j - \underline{T}_j$ as the gap for the parameter bounds. The average gap for each facility among the 432 instances is shown in Table 2. We find the bounds are quite tight on average; in some cases we have $\bar{T}_j = \underline{T}_j$, which directly gives the optimal T_j^*.

To further analyze the impact of system parameters on the effectiveness of bounds, we conduct a parametric analysis on the system parameters. Table 3 summaries the result.

We observe that Δ_j tends to be increasing in K_j and decreasing in h_j and λ_j. For example, when K_2 increases from 8 to 32, Δ_2 increases from 4.9 to 8.8, and when λ_2 increases from 3 to 6, Δ_2 decreases from 7.3 to 4.1. We also observe that Δ_j is relatively insensitive to the change of facility i's parameters, where $i \neq j$. For example, when K_0

Table 2. Summary for the gap of parameter bounds for $N = 2$.

	Δ_0	Δ_1	Δ_2
Max	11	21	28
Min	0	1	0
Average	3.2	6.8	5.7

Table 3. Impact of parameters on the gap of the parameter bounds.

K_0	$(\Delta_0, \Delta_1, \Delta_2)$	K_1	$(\Delta_0, \Delta_1, \Delta_2)$	K_2	$(\Delta_0, \Delta_1, \Delta_2)$
2	(2.4, 7.3, 6.4)	2	(2.8, 3.9, 5.2)	2	(3.1, 6.6, 3.4)
8	(3.0, 6.4, 5.5)	8	(2.7, 5.7, 5.1)	8	(2.9, 6.2, 4.9)
32	(4.1, 6.5, 5.3)	32	(4.0, 10.7, 6.8)	32	(3.5, 7.5, 8.8)
h_0	$(\Delta_0, \Delta_1, \Delta_2)$	h_2	$(\Delta_0, \Delta_1, \Delta_2)$	λ_2	$(\Delta_0, \Delta_1, \Delta_2)$
0.5	(4.2, 4.6, 3.9)	0.5	(3.0, 6.8, 9.0)	3	(3.7, 6.3, 7.3)
2	(2.1, 8.9, 7.6)	2	(3.3, 6.7, 2.5)	6	(2.6, 7.2, 4.1)

Table 4. Summary for the average gap of the parameter bounds for four- and eight-retailer systems.

N	Δ_0	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8
4	3.4	8.1	7.1	7.7	8.5	—	—	—	—
8	2.5	13.6	11.5	12.1	12.9	12.9	11.5	12.1	12.9

increases from 2 to 32, Δ_2 only changes from 6.4 to 5.3. Overall, the gap for the retailers is larger and more sensitive to the change of parameters than that of the warehouse. For example, Δ_0 changes from 4.2 to 2.1 when h_0 increases from 0.5 to 2, whereas Δ_2 changes from 9.0 to 2.5 when h_2 increases from 0.5 to 2.

We provide an explanation of why the gap of the parameter bounds of retailer j increases in K_j and decreases in h_j and λ_j and consequently increases in the ratio of $K_j/(h_j\lambda_j)$. Recall that the bounds are obtained by using $\tilde{c}_j(T_j)$ to search over T_j within the region where $\tilde{c}_j(T_j)$ is less than the cost difference, $(C^h - \sum_{i\neq j} \underline{c}_i)$. Thus, if the cost difference becomes larger, the gap between the parameter bounds becomes bigger. Intuitively, when the cost ratio $K_j/(h_j\lambda_j)$ is larger, the corresponding reorder interval tends to be longer, making the resulting cost $\underline{c}_j$ bigger. This implies that the cost difference $(C^h - \sum_{i\neq j} \underline{c}_i)$ will be bigger as well.

We have evaluated the parameter bounds for instances with $N = 4$ and $N = 8$. The parameters are the same as those of the instances in the previous subsection. Table 4 shows the average gap of the parameter bounds $(\bar{T}_j - \underline{T}_j)$ for each retailer j. As expected, the gap increases in the number of retailers. Thus, our proposed heuristic is particularly useful when solving larger systems.

6. Concluding Remarks

This paper studies a stochastic, one-warehouse-multi-retailer system in which an echelon (S, T) policy is implemented. When the demand is deterministic, it is known that there exists a simple POT policy that yields a near-optimal cost. However, fewer results are known for the corresponding stochastic model. In this paper, we first derive a simple bottom-up recursion to evaluate the system cost for any given (S, T) policy. We then construct a series of lower bound cost functions that enable a method of obtaining the optimal reorder intervals and an efficient heuristic. In the numerical study, we find that the optimal reorder intervals tend to represent an integer-ratio policy and the deterministic POT policy may be ineffective, under some conditions.

Our model assumes that the warehouse implements the virtual allocation rule. It is interesting to extend the current analysis to other allocation policies. (In general, it would be very difficult to characterize the exact optimal policy for the distribution system, which depends on the inventory states of facilities.) We also assume deterministic lead times in our model. This assumption facilitates the synchronized scheduling rule. It remains an open question of what would be a plausible and effective scheduling rule for the system with stochastic lead times. We leave these studies for future research.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2015.1347.

Acknowledgments

The authors would like to thank Professor Izak Duenyas, the associate editor, and anonymous reviewers for their helpful comments. The third author is partially supported by Hong Kong General Research Fund [CUHK-419010] and the Asian Institute of Supply Chains and Logistics at CUHK.

Appendix A. Optimization of Base-Stock Levels with Fixed T

This appendix shows how to optimize the optimal echelon base-stock levels when the reorder intervals $\mathbf{T}$ are fixed. It is worth mentioning that Axsäter (1993) provides an approach (which is a generalization of Axsäter (1990) on base-stock systems) for finding the optimal base-stock levels with fixed reorder intervals. Axsäter's model is a special case of ours because he assumes that the retailers have the same reorder interval and that the reorder interval of the warehouse is an integer multiple of those of the retailers (i.e., the so-called nested integer-ratio policies). Below we scratch the idea of how his approach can be generalized to our model. The proofs for Propositions 7–9 are shown in Appendix B.

We present the analysis from the local policy perspective because B_0 is a function of s_0 and S_j is the same as the local base-stock levels s_j.

Define the local inventory holding and backorder cost for retailer j as follows:

$$f_j(y,r) = \mathsf{E}\big[H_j(y - D_j[L_j + \mathbb{M}_{T_j}(r)]) + (b_j + H_j)(y - D_j[L_j + \mathbb{M}_{T_j}(r)])^-\big], \quad j = 1, \ldots, N,$$

and

$$f_0(y,r) = h_0\,\mathsf{E}\bigg[y - D_0[L_0 + \mathbb{M}_{T_0}(r)] + \sum_{j=1}^{N} D_j[L_j + \mathbb{M}_{T_j}(r)] + \sum_{j=1}^{N} \frac{\lambda_j}{\lambda_0}(y - D_0[L_0 + M_{T_0}(r_j(r))])^-\bigg].$$

Proposition 7.

$$C(\mathbf{S},\mathbf{T}) = \sum_{j=0}^{N} \frac{K_j}{T_j} + \frac{1}{T}\sum_{r=0}^{T-1}\bigg(f_0(s_0,r) + \sum_{j=1}^{N} \mathsf{E}[f_j(s_j - B_{0j}(L_0 + r_j(r)), r)]\bigg) + h_0 \sum_{j=1}^{N} (\lambda_j L_j).$$

Note that $f_0(s_0, r)$ is the warehouse inventory holding cost; $f_j(s_j - B_{0j}(L_0 + r_j(r)), r)$ is retailer j's inventory holding and backorder cost; and the last term is the average holding cost of pipeline inventory per period, which is constant.

For convenience, let us define

$$\hat{f}_0(s_0) = \frac{1}{T}\sum_{r=0}^{T-1} f_0(s_0, r),$$

$$\hat{f}_j(s_0, s_j) = \frac{1}{T}\sum_{r=0}^{T-1} \mathsf{E}\big[f_j(s_j - B_{0j}(L_0 + r_j(r)), r)\big].$$

Here, $\hat{f}_j(\cdot,\cdot)$ is a function of s_0 because B_{0j} is a function of s_0.

Proposition 8. *For fixed* $\mathbf{T}$ *and* s_0, $\hat{f}_j(s_0, s_j)$ *is convex in* s_j.

With Proposition 8, we can find the best local base-stock level $s_j(s_0)$ for each retailer j. That is,

$$s_j(s_0) = \arg\min_{s_j} \hat{f}_j(s_0, s_j).$$

Substituting $s_j(s_0)$ for s_j in $C(\mathbf{S},\mathbf{T})$, the objective function becomes a function of s_0; i.e.,

$$C(s_0) = \hat{f}_0(s_0) + \sum_{j=1}^{N} \hat{f}_j(s_0, s_j(s_0)).$$

Unfortunately, $C(s_0)$ is not convex in s_0, so we have to construct bounds for the optimal s_0, denoted as s_0^*, and conduct a search over the feasible interval.

Following Axsäter's (1990) approach, we next provide bounds for s_0^*. Let $s_j^l = s_j(\infty)$ and $s_j^u = s_j(0)$. Define

$$s_0^u = \arg\min_{s_0}\left\{\hat{f}_0(s_0) + \sum_{j=1}^{N}\hat{f}_j(s_0, s_j^l)\right\}$$

and

$$s_0^l = \arg\min_{s_0}\left\{\hat{f}_0(s_0) + \sum_{j=1}^{N}\hat{f}_j(s_0, s_j^u)\right\}.$$

Then,

Proposition 9. (1) $s_j^l \leqslant s_j^* \leqslant s_j^u$; (2) $s_0^l \leqslant s_0^* \leqslant s_0^u$.

With Proposition 9, we can search over all possible s_0 between s_0^l and s_0^u. After s_0^* is found, the optimal base-stock level for retailer j is $s_j^* = s_j(s_0^*)$.

Appendix B. Proofs

Proposition 2

Note that

$$\begin{aligned}\underline{g}_0(S_0, T_0) &= \frac{1}{T_0}\sum_{r=0}^{T_0-1}\mathsf{E}\Bigg[h_0\,\mathrm{IL}_0(t+L_0+r) + \min_{\sum_j y_j(r)\leqslant \mathrm{IL}_0^-(t+L_0+r)}\sum_{j=1}^{N} g_{j0}^0(y_j(r))\Bigg]\\ &= \frac{1}{T_0}\sum_{r=0}^{T_0-1}\mathsf{E}\Bigg[h_0(S_0 - D[L_0+r]) + \min_{\sum_j y_j(r)\leqslant S_0 - D[L_0+r-1]}\sum_{j=1}^{N} g_{j0}^0(y_j(r))\Bigg],\end{aligned}$$

where the first term in the brackets is clearly convex in S_0; the second term is also convex in S_0 because with any realized $D[L_0 + r-1]$, it is the minimum of a summation of convex functions subject to a linear constraint (that depends on S_0). Thus $\underline{g}_0(S_0, T_0)$ is convex in S_0 for fixed T_0.

Proposition 5

To show (1), we see that for $j = 1, \ldots, N$,

$$\begin{aligned}\underline{c}_j(T_j) = \frac{K_j}{T_j} + \min_y \frac{1}{T_j} &\cdot \mathsf{E}\sum_{r=0}^{T_j-1}\big[h_{jj}(y - D_j[L_j+r]) + (h_{jj}+b_{jj})(y - D_j[L_j+r])^-\big]\\ &+ g_{jj}^0(S_j).\end{aligned}$$

For fixed T_j and y, $K_j/T_j + (1/T_j)\mathsf{E}\sum_{r=0}^{T_j-1}[h_{jj}(y - D_j[L_j+r]) + (h_{jj}+b_{jj})(y - D_j[L_j+r])^-] + g_{jj}^0(S_j)$ is convex in $D_j[L_j+r]$; therefore, by Jensen's inequality,

$$\begin{aligned}&\frac{K_j}{T_j} + \frac{1}{T_j}\mathsf{E}\sum_{r=0}^{T_j-1}\big[h_j^j(y - D_j[L_j+r]) + (h_j^j + b_j^j)(y - D_j[L_j+r])^-\big] + g_{jj}^0(S_j)\\ &\geqslant \frac{K_j}{T_j} + \frac{1}{T_j}\sum_{r=0}^{T_j-1}\big[h_j^j(y - \mathsf{E}D_j[L_j+r]) + (h_j^j+b_j^j)(y - \mathsf{E}D_j[L_j+r])^-\big] + g_{jj}^0(S_j)\\ &= \frac{K_j}{T_j} + \frac{1}{T_j}\sum_{r=0}^{T_j-1}\big[h_j^j(y - \lambda_j(L_j+r+1)) + (h_j^j+b_j^j)(y - \lambda_j(L_j+r+1))^-\big] + g_{jj}^0(S_j).\end{aligned}$$

Optimizing over y on both sides of the above inequality will lead to $\underline{c}_j(T_j) \geqslant \tilde{c}_j(T_j)$. Similar arguments apply to $\underline{c}_0(T_0)$.

To prove (2), we will show that for any continuous convex function $f(x)$: $\mathbb{R}\to\mathbb{R}$,

$$h(T) = K/T + \min_x (1/T)\sum_{r=0}^{T-1} f(x - ry)\colon \mathbb{Z}^+ \to \mathbb{R}$$

is quasiconvex in T for any $y > 0$ and $K > 0$. We see that $\tilde{c}_j(T_j)$ can be represented in such a form. Let

$$x_1 = \arg\min_x \sum_{r=0}^{T-2} f(x - ry),$$

$$x_2 = \arg\min_x \sum_{r=0}^{T-1} f(x - ry),$$

$$x_3 = \arg\min_x \sum_{r=0}^{T} f(x - ry),$$

By the optimality of x_3, we must have $f'(x_3 - Ty) \leqslant 0$; otherwise, $f'(x_3 - ry) \geqslant f'(x_3 - Ty) > 0$ for all $r = 0, \ldots, T$ and therefore $\sum_{r=0}^{T} f'(x_3 - ry) > 0$, which contradicts the optimality of x_3. Similarly, we must have $f'(x_3) \geqslant 0$, $f'(x_2) \geqslant 0$, $f'(x_1) \geqslant 0$ and $f'(x_2 - (T-1)y) \leqslant 0$, $f'(x_1 - (T-2)y) \leqslant 0$.

It then follows that

$$\begin{aligned}&h(T+1) - h(T)\\ &= \frac{K}{T+1} + \frac{1}{T+1}\sum_{r=0}^{T} f(x_3 - ry) - \frac{K}{T} - \frac{1}{T}\sum_{r=0}^{T-1} f(x_2 - ry)\\ &= \frac{1}{T(T+1)}\Bigg[T\sum_{r=0}^{T} f(x_3 - ry) - (T+1)\sum_{r=0}^{T-1} f(x_2 - ry) - K\Bigg],\end{aligned}$$

and

$$h(T)-h(T-1)=\frac{1}{T(T-1)}\left[(T-1)\sum_{r=0}^{T-1}f(x_2-ry)-T\sum_{r=0}^{T-2}f(x_1-ry)-K\right].$$

We only need to show that

$$T\sum_{r=0}^{T}f(x_3-ry)-(T+1)\sum_{r=0}^{T-1}f(x_2-ry)\geqslant (T-1)\sum_{r=0}^{T-1}f(x_2-ry)-T\sum_{r=0}^{T-2}f(x_1-ry),$$

or equivalently

$$\sum_{r=0}^{T}f(x_3-ry)+\sum_{r=0}^{T-2}f(x_1-ry)\geqslant 2\sum_{r=0}^{T-1}f(x_2-ry). \tag{B1}$$

If (B1) is true, then it implies that if $h(T)\geqslant h(T-1)$, then $h(T+1)\geqslant h(T)$; i.e., $h(T)$ is quasiconvex.

If

$$f(x_3-Ty)\geqslant f(x_1-(T-1)y),$$

then

$$\sum_{r=0}^{T}f(x_3-ry)+\sum_{r=0}^{T-2}f(x_1-ry)\geqslant\sum_{r=0}^{T-1}f(x_3-ry)+\sum_{r=0}^{T-1}f(x_1-ry)\geqslant 2\sum_{r=0}^{T-1}f(x_2-ry),$$

where the second inequality is due to the optimality of x_2. On the other hand, if

$$f(x_3-Ty)<f(x_1-(T-1)y),$$

then $x_3-Ty\geqslant x_1-(T-1)y$ because $f'(x_1-(T-1)y)\leqslant f'(x_1-(T-2)y)\leqslant 0$ (see the discussion after the definition of x_1, x_2, x_3). Then we have

$$x_3\geqslant x_1+y.$$

Furthermore, because $f'(x_1+y)\geqslant f'(x_1)\geqslant 0$, $f(x_3)\geqslant f(x_1+y)$. Henceforth

$$\sum_{r=0}^{T}f(x_3-ry)+\sum_{r=0}^{T-2}f(x_1-ry)\geqslant\sum_{r=1}^{T}f(x_3-ry)+\sum_{r=-1}^{T-2}f(x_1-ry)=\sum_{r=0}^{T-1}f(x_3-y-ry)+\sum_{r=0}^{T-1}f(x_1+y-ry)\geqslant 2\sum_{r=0}^{T-1}f(x_2-ry),$$

where the second inequality follows again from the optimality of x_2. Therefore, we have proved that $h(T)$ is quasiconvex in $T\in\mathbb{Z}^+$.

Proposition 6

Let the optimal reorder intervals be $(T_0, T_1, \ldots, T_N)$. Given T_i satisfies integer ratio relations, we want to show $T_0\geqslant\min_{j\geqslant 1}\{T_j\}$. Consider two retailers with $T_1\leqslant T_2$. Due to integer ratio constraint, $\mathbb{M}_{T_1}(T_2)=0$. Suppose $T_0<T_1$ and so $\mathbb{M}_{T_0}(T_1)=0$ and denote $n_1=T_1/T_0$. And it can be seen that $M=T_2$ by its definition. We compare the costs of two systems: one with (T_0, T_1, T_2) and the other with (T_1, T_1, T_2). (S_0, S_1, S_2) are given echelon base-stock levels.

For $r=0,1,\ldots,T_2-1$, we first study how $B_0(t+L_0+r_j(r))$ differs in these two systems for all $j\geqslant 1$. Recall that

$$B_0(t+L_0+r_j(r))=\left[s_0-D_0[t+r_0(r_j(r)), t+L_0+r_j(r))\right]^-.$$

For $j=2$, it is clear that with given s_0, $B_0(t+L_0+r_j(r))$ is the same in both systems as $r_j(r)=0$. Thus we just consider $j=1$. When the warehouse reorder interval is T_1, $r_0(r_1(r))=\lfloor r/T_1\rfloor T_1$; when its reorder interval is T_0,

$$r_0(r_1(r))=\left\lfloor\frac{\lfloor r/T_1\rfloor T_1}{T_0}\right\rfloor T_0=n_1\left\lfloor\frac{r}{T_1}\right\rfloor T_0=\left\lfloor\frac{r}{T_1}\right\rfloor T_1$$

since $\mathbb{M}_{T_0}(T_1)=0$ and $n_1=T_1/T_0$. Thus with given s_0, these two systems share the same $B_0(t+L_0+r_j(r))$ for each r and so the same $B_{0j}(t+L_0+r_j(r))$.

The system inventory related cost is evaluated as

$$\frac{1}{M}\sum_{r=0}^{M-1}G_0(\mathbf{S},\mathbf{T},r)=\frac{1}{M}\sum_{r=0}^{M-1}\mathsf{E}\Big[h_0(S_0-D_0[L_0+\mathbb{M}_{T_0}(r)])+\sum_{j=1}^{N}G_j(S_j-B_{0j}(L_0+r_j(r)), T_j, r)\Big],$$

where

$$G_j(y,T_j,r)=\mathsf{E}\big[h_j(y-D_j[L_j+\mathbb{M}_{T_j}(r)])+(b_j+H_j)(y-D_j[L_j+\mathbb{M}_{T_j}(r)])^-\big].$$

It can be seen that if one increases T_0 to T_1, both M and $\mathsf{E}[\sum_{j=1}^N G_j(S_j-B_{0j}(L_0+r_j(r)), T_j, r)]$ remain unchanged. However, the average fixed cost at the warehouse decreases and the holding cost term of the warehouse also decreases as $D_0[L_0+\mathbb{M}_{T_0}(r)]$ increases. Hence, the system with (T_0, T_1, T_2) incurs a higher cost than does the system with (T_1, T_1, T_2) that uses the same base-stock levels. Therefore, the result holds for a two-retailer system. The proof can be readily extended to a system with general multiple retailers, so we omit the details.

Proposition 7

Recall that

$$G_j(y,T_j,r)=\mathsf{E}\big[h_j(y-D_j[L_j+\mathbb{M}_{T_j}(r)])+(b_j+H_j)(y-D_j[L_j+\mathbb{M}_{T_j}(r)])^-\big].$$

and

$$G_0(\mathbf{S},\mathbf{T},r)=\mathsf{E}\Big[h_0(S_0-D_0[L_0+\mathbb{M}_{T_0}(r)])+\sum_{j=1}^{N}G_j(S_j-B_{0j}(L_0+r_j(r)), r)\Big]$$

Because $s_j = S_j$ and $s_0 = S_0 - \sum_{j=1}^N S_j$,

$$G_0(\mathbf{S}, \mathbf{T}, r) = \mathsf{E}\big[h_0(S_0 - D_0[L_0 + \mathbb{M}_{T_0}(r)])\big] + \sum_{j=1}^N \mathsf{E}\big[h_j(s_j - B_{0j}(L_0 + r_j(r)) - D_j[L_j + \mathbb{M}_{T_j}(r)]) + (b_j + H_j)(s_j - B_{0j}(L_0 + r_j(r)) - D_j[L_j + \mathbb{M}_{T_j}(r)])^-\big]$$

$$= \mathsf{E}\big[h_0(S_0 - D_0[L_0 + \mathbb{M}_{T_0}(r)])\big] - \sum_{j=1}^N \mathsf{E}\big[h_0(s_j - B_{0j}(L_0 + r_j(r)) - D_j[L_j + \mathbb{M}_{T_j}(r)])\big] + \sum_{j=1}^N \mathsf{E}\big[H_j(s_j - B_{0j}(L_0 + r_j(r)) - D_j[L_j + \mathbb{M}_{T_j}(r)]) + (b_j + H_j)(s_j - B_{0j}(L_0 + r_j(r)) - D_j[L_j + \mathbb{M}_{T_j}(r)])^-\big]$$

$$= \mathsf{E}\big[h_0(s_0 - D_0[L_0 + \mathbb{M}_{T_0}(r)])\big] + \sum_{j=1}^N \mathsf{E}\big[h_0(B_{0j}(L_0 + r_j(r)))\big] - \sum_{j=1}^N \mathsf{E}\big[h_0(-D_j[L_j + \mathbb{M}_{T_j}(r)])\big] + \sum_{j=1}^N \mathsf{E}\big[f_j(s_j - B_{0j}(L_0 + r_j(r)), r)\big]$$

$$= h_0\,\mathsf{E}\Big[\Big(s_0 - D_0[L_0 + \mathbb{M}_{T_0}(r)] + \sum_{j=1}^N (D_j[\mathbb{M}_{T_j}(r)])\Big) + \sum_{j=1}^N \frac{\lambda_j}{\lambda_0}\big(s_0 - D_0[L_0 + r_j(r) - r_0(r_j(r))]\big)^-\Big] + \sum_{j=1}^N h_0(\lambda_j(L_j)) + \sum_{j=1}^N \mathsf{E}\big[f_j(s_j - B_{0j}(L_0 + r_j(r)), r)\big].$$

Because $h_0 = H_0$, so from Proposition 1, the result follows.

Proposition 8

The convexity on s_j follows directly from the definition of $\hat{f}_j(s_0, s_j)$. We omit the detailed proof for brevity.

Proposition 9

We need to first show that $\hat{f}_j(s_0, s_j)$ is supermodular in s_0 and s_j, or

$$\hat{f}_j(s_0, s_j + 1) - \hat{f}_j(s_0, s_j) \leqslant \hat{f}_j(s_0 + 1, s_j + 1) - \hat{f}_j(s_0 + 1, s_j). \tag{B2}$$

For notational simplicity, we denote $D_j[L_j + \mathbb{M}_{T_j}(r)]$ by D_j. In addition, to emphasize the dependency of B_{0j} on s_0 and for brevity, let $B_{0j}(s_0)$ denote $B_{0j}(L_0 + r_j(r))$. Note that

$$\hat{f}_j(s_0, s_j + 1) - \hat{f}_j(s_0, s_j) = H_j + \frac{1}{T}\sum_{r=0}^{T-1}(b_j + H_j)\,\mathsf{E}\big[(s_j + 1 - B_{0j}(s_0) - D_j)^- - (s_j - B_{0j}(s_0) - D_j)^-\big]$$

$$\leqslant H_j + \frac{1}{T}\sum_{r=0}^{T-1}(b_j + H_j)\,\mathsf{E}\big[(s_j + 1 - B_{0j}(s_0 + 1) - D_j)^- - (s_j - B_{0j}(s_0 + 1) - D_j)^-\big]$$

$$= \hat{f}_j(s_0 + 1, s_j + 1) - \hat{f}_j(s_0 + 1, s_j)$$

because $()^-$ is convex and $B_{0j}(s_0 + 1)$ is smaller than $B_{0j}(s_0)$. Thus the inequality (B2) is valid. With this result and the convexity of $\hat{f}_j(s_0, s_j)$ is s_j, part (1) follows.

To show part (2), from (B2), we have, for any $s_j \geqslant s_j^l$,

$$0 \geqslant \hat{f}_0(s_0^u) + \sum_{j=1}^N \hat{f}_j(s_j^l, s_0^u) - \Big[\hat{f}_0(s_0^u + 1) + \sum_{j=1}^N \hat{f}_j(s_j^l, s_0^u + 1)\Big]$$

$$\geqslant \hat{f}_0(s_0^u) + \sum_{j=1}^N \hat{f}_j(s_j, s_0^u) - \Big[\hat{f}_0(s_0^u + 1) + \sum_{j=1}^N \hat{f}_j(s_j, s_0^u + 1)\Big]$$

where the first inequality follows from the definition of s_0^u. Because the optimal s_j^* will be greater than s_j^l, s_0^u is an upper bound for s_0^*. Similarly, we can verify the lower bound s_0^l.

Endnotes

1. The virtual allocation rule is commonly seen in practice. For example, Wal-Mart's distribution center assigns replenishment stocks to the demands as they occur. The assigned stocks are loaded onto a truck and shipped to the retail stores according to some fixed schedule (Chandran 2003). This rule is essentially the first-come-first-serve rule. We refer the authors to Graves (1996) for a detailed discussion of its applications.
2. For simplicity, we assume that the facilities will place an order at each order epoch. If the fixed order cost is incurred only when a facility places an order, the fixed cost term should be modified as $\sum_{j=0}^N (K_j \Pr(D[T_j] > 0)/T_j)$. This addition will not affect the algorithm of finding the optimal base-stock level **S** for fixed **T**. Following Shang and Zhou (2010, §5), it can be shown that our approach of finding the optimal reorder intervals **T** in §4 can be carried over to this alternative cost expression.
3. These 128 instances are generated from the following parameters: We fix retailer 1's parameters and vary the parameters for the warehouse and retailer 2. More specifically, the parameters for retailer 1 are $K_1 = h_1 = L_1 = \lambda_1 = 1$ and $b_1 = 3$. For retailer 2, we set $h_2 = 1$. The rest of the parameters for the warehouse and retailer 2 are chosen from the following sets: $h_0 \in \{1, 2\}$, K_0, $K_2 \in \{0.25, 16\}$, L_0, $L_2 \in \{1, 3\}$, $b_2 \in \{3, 18\}$, $\lambda_2 \in \{0.5, 1\}$.
4. Notice that $\sum_{j\in E} \underline{c}_j(T)$ and $\underline{c}_j(T)$ for $j \notin E$ may not be convex in T. In our numerical study, these cost functions tend to have a quasiconvex shape, and we therefore set the solution to be the first minimizer of these cost functions.
5. The average running time for finding optimal solution for $N = 2, 4$, and 8 is more than 0.6, 2, and 7 hours, respectively.

References

Atkins D, Iyogun P (1988) Period versus "can-order" policies for coordinated multi-item inventory systems. *Management Sci.* 34(6):791–796.
Axsäter S (1990) Simple solution procedure for a class of two-echelon inventory problems. *Oper. Res.* 38(1):64–69.
Axsäter S (1993) Optimization of order-up-to-*S* policies in two-echelon inventory systems with periodic review. *Naval Res. Logist.* 40: 245–253.
Cachon G (1999) Managing supply chain demand variability with scheduled ordering policies. *Management Sci.* 45(6):843–856.

Çetinkaya S, Lee C-Y (2000) Stock replenishment and shipment scheduling for vendor-managed inventory systems. *Management Sci.* 46(2):217–232.

Chandran M (2003) Wal-Mart's supply chain management practice. ICFAI Center for Management Research, Nagarjuna Hills, Hyderabad, India.

Chen F, Samroengraja R (2004) Order volatility and supply chain costs. *Oper. Res.* 52(5):707–722.

Chen F, Zheng Y-S (1994) Lower bounds for multi-echelon stochastic inventory systems. *Management Sci.* 40(11):1426–1443.

Chen F, Zheng Y-S (1997) One-warehouse multi-retailer systems with centralized stock information. *Oper. Res.* 45(2):275–287.

Chen F, Zheng Y-S (1998) Near-optimal echelon-stock (R, nQ) policies in multistage serial systems. *Oper. Res.* 46(4):592–602.

Cheung K-L, Zhang S-H (2008) Balanced and synchronized ordering in supply chains. *IIE Trans.* 40:1–11.

Chu L, Shen Z-J (2010) A power-of-two ordering policy for one-warehouse multiretailer systems with stochastic demand. *Oper. Res.* 58(2):492–502.

Clark A, Scarf H (1960) Optimal policies for a multi-echelon inventory problem. *Management Sci.* 6(4):475–490.

Federgruen A, Groenevelt H, Tijms H (1984) Coordinated replenishments in a multi-item inventory system with compound Poisson demands. *Management Sci.* 30(3):344–357.

Graves S (1996) A multiechelon inventory model with fixed replenishment intervals. *Management Sci.* 42(1):1–18.

Gürbüz M, Moinzadeh K, Zhou Y-P (2007) Coordinated replenishment strategies in inventory/distribution systems. *Management Sci.* 53(2):293–307.

Lee H, Padmanabhan V, Whang S (1997) Information distortion in a supply chain: The bullwhip effect. *Management Sci.* 43(4):546–558.

Liu F, Song J-S (2012) Good and bad news about the (S, T) policy. *Manufacturing Service Oper. Management* 14(1):42–49.

Marklund J (2011) Inventory control in divergent supply chains with time based dispatching and shipment consolidation. *Naval Res. Logist.* 58:59–71.

Maxwell W, Muckstadt J (1985) Establishing consistent and realistic reorder intervals in production-distribution systems. *Oper. Res.* 33(6): 1316–1341.

Rao US (2003) Properties of the period review (R, T) inventory control policy for stationary, stochastic demand. *Manufacturing Service Oper. Management* 5(1):37–53.

Roundy R (1985) 98% effective integer-ratio lot sizing for one-warehouse multi-retailer systems. *Management Sci.* 31(11):1416–1430.

Schwarz L (1973) A simple continuous review deterministic one-warehouse N-retailer inventory problem. *Management Sci.* 19(5):555–566.

Shang K, Zhou S (2010) Optimal and heuristic (r, nQ, T) policies in serial inventory systems with fixed costs. *Oper. Res.* 58(2):414–427.

Silver E (1981) Establishing reorder points in the (S, c, s) coordinated control system under compound Poisson demand. *Internat. J. Production Res.* 9:743–750.

Simon RM (1981) Stationary properties of a two-echelon inventory model for low demand items. *Oper. Res.* 19(3):761–777.

Kevin Shang is an associate professor in the Fuqua School of Business, Duke University. His research mainly focuses on developing simple and effective inventory policies for supply chains. He is also interested in issues related to the interface of operations and finance and corporate sustainability.

Sean X. Zhou is an associate professor in the department of decision sciences and managerial economics, CUHK Business School, the Chinese University of Hong Kong. He is also the director of the Supply Chain Research Center under the Asian Institute of Supply Chain and Logistics in CUHK. His main research area is supply chain management with particular interests in inventory control, production planning, pricing, and game theoretic applications.

Zhijie Tao is an assistant professor of operations management at the Shanghai University of Finance and Economics. His research interest is supply chain management and inventory management.