Investments in Renewable and Conventional Energy: The Role of Operational Flexibility

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There is an ongoing debate on how providing a subsidy for one energy source affects the investment level of other sources. To investigate this issue, we study a capacity investment problem for a utility firm that invests in renewable and conventional energy, with a consideration of two critical factors. First, conventional sources have different levels of operational flexibility, i.e., inflexible (e.g., nuclear and coal) and flexible (e.g., natural gas). Second, random renewable energy supply and electricity demand are correlated and nonstationary. We model this problem as a two-stage stochastic program in which a utility firm first determines the capacity investment levels, followed by the dispatch quantities of energy sources, to minimize the sum of investment and generation-related costs. We derive the optimal capacity portfolio to characterize the interactions between renewable and conventional sources. We find that renewable and inflexible sources are substitutes, suggesting that a subsidy for nuclear or coal-fired power plants leads to a lower investment level in wind or solar energy. On the other hand, wind energy and flexible sources are complements. Thus, a subsidy for flexible natural gas-fired power plants leads to a higher investment in wind energy. This result holds for solar energy if the subsidy for the flexible source is sufficiently high. We validate these insights by using real electricity generation and demand data from the state of Texas.

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1 Introduction

Policymakers have introduced various subsidies to encourage investment in clean energy sources in order to reduce carbon emissions. For instance, the U.S. government provides a 30% subsidy for investment costs in solar energy (SEIA 2016), and the state of New York is planning to offer a multibillion-dollar subsidy for nuclear power plants (Yee 2016). However, there is an ongoing debate on how an increased investment in one energy source (because of a subsidy) affects the investment in other energy sources. On the one hand, Dotson (2013) explains that renewables are supported by nuclear power because nuclear can generate steady electricity to supplement intermittent renewables. On the other hand, the former chairman of the Federal Energy Regulatory Commission states that no new nuclear investment is needed in the presence of increased renewable investment because nuclear power is inflexible, that is, a nuclear plant cannot be ramped up or down quickly (Straub and Behr 2009). Contradictory claims are also reported on the interaction between renewable and natural gas-fired power plants. In The New York Times, Kotchen (2012) claims that low natural gas prices are a "trap" for renewables because, in response to the lower natural gas cost, a utility firm would invest more in natural gas-fired plants than in renewables. On the contrary, in The Wall Street Journal, Keith (2013) calls this claim a "myth" related to renewables and explains that natural gas can complement renewables by alleviating the intermittency problem. In this paper, we investigate these interactions between energy sources by focusing on the capacity investments of utility firms, which undertake the majority of the energy investments in the U.S.

In recent years, utility firms have significantly invested in renewable sources, such as solar and wind energy, because these sources provide electricity with negligible generation costs. Utility firms also invest in conventional sources, which are categorized into two groups—inflexible and flexible—based on operational flexibility, that is, whether the output of the source can be ramped up or down quickly. A nuclear or coal-fired power plant is inflexible because its output cannot be changed rapidly due to technical reasons. A combined-cycle natural gas-fired power plant is also relatively inflexible. On the other hand, open-cycle natural gas- or oil-fired power plants are flexible (DOE 2011). From the cost perspective, an inflexible source has higher investment but lower generation (fuel) costs than a flexible source. Considering these characteristics, it is challenging for a utility firm to determine the right capacity investment portfolio that minimizes its investment

and generation costs while maintaining a certain reliability level (i.e., the chance of no blackouts). For example, Smith (2013) has identified a "looming energy crisis" for utility firms in California because they do not have "the right mix of power plants" and are vulnerable to reliability problems because of over-reliance on intermittent renewables. Motivated by these policy discussions, we pose the following research questions: What capacity portfolio for a utility firm minimizes the investment and generation costs in the presence of inflexible, renewable, and flexible sources? What is the role of operational flexibility in the interaction between conventional and renewable sources? How does a carbon tax policy affect energy investments?

We model this problem following the decision process of a utility firm for making capacity investments. Specifically, a utility firm first takes a long-term, strategic capacity decision by investing in different energy sources. The invested capacity level of a source is the maximum output that the utility firm can dispatch from that source during each of the operating periods, which is often set to be five minutes. The decision of dispatching the electricity supply to match the demand is based on five-minute-ahead forecasts of the electricity demand and the intermittency of renewable sources. If the demand cannot be satisfied, a penalty cost is incurred. This penalty cost represents consumers' inconvenience costs and the utility firm's energy procurement cost from external sources. One challenge of this problem is that the random electricity demand and renewable energy supply are not only correlated in a given period, but they are also serially correlated and nonstationary over time. We take these into account and formulate the problem as a two-stage stochastic program with recourse. In the first stage, under the joint distribution of demand and supply uncertainties, the firm makes a strategic decision by determining the capacity investment in the inflexible, renewable, and flexible sources. In the second stage, the firm determines the amount of electricity dispatched from these energy sources for each operating period based on the forecasts. The objective of the utility firm is to minimize the total expected cost, which is the sum of the initial investment costs, the electricity generation costs, and the penalty costs of supply shortage.

We solve the utility firm's investment problem by using backward induction and characterize the optimal dispatch policy: all inflexible capacity is first used, followed by the renewable energy capacity, as its generation cost is negligible compared to the flexible source, which is used as the last resort. Based on this optimal dispatch policy, we determine the optimal investment level for each source. We obtain a multi-dimensional newsvendor-type solution. That is, the utility firm

balances the underage cost (e.g., the penalty cost due to supply shortage) with the overage cost (i.e., the investment cost) for each energy source in the demand and intermittency space. In the most practical case in which the investment levels of all sources are positive, the critical fractile associated with the flexible source determines the probability of meeting the demand. This indicates that the reliability of the electricity system (proxied by the loss-of-load probability, see Corollary 1) is determined by the cost parameters of the flexible source and the penalty cost rate. This finding reveals an important policy insight that the reliability is only affected by a subsidy provided for the flexible source but not the subsidies for renewable and inflexible sources.

To identify how a subsidy for one source affects the investment level of the other sources, we examine the interaction between energy sources. Specifically, we define two sources as *substitutes* (*complements*, respectively) if a decrease in the investment cost of one source leads to a decrease (an increase, respectively) in the investment level of the other. One might think that energy sources are substitutes with each other because they jointly satisfy the demand. Interestingly, we show that renewable and flexible sources are complements under certain conditions. This result is due to operational flexibility. Specifically, increased investment in the flexible source (due to a subsidy) enables the utility firm to adjust its energy output quickly, which alleviates the intermittency problem. Consequently, the utility firm also increases the renewable investment to take advantage of its negligible generation cost. This effect is particularly strong for wind energy because a higher output from the flexible source satisfies the high demand during daytime when the wind output tends to be low. On the other hand, renewable and inflexible sources are substitutes. We verify these analytical results in a case study based on real electricity generation and demand data from Texas in Section 6, and we find that the complementarity effect also holds for solar energy.

We also consider the effect of a carbon tax on energy investments. Many experts claim that taxing carbon emissions motivates investment in renewable sources (c.f., Caperton 2012 and Porter 2014). Our analysis indicates that this claim does not hold if the inflexible source is carbon-free nuclear energy. In this case, the carbon tax only increases the generation cost of the flexible source. This results in a reduction in the investment of the flexible source, which, in turn, reduces the investment of the renewable source due to the complementarity effect.

The rest of the paper is organized as follows. Section 2 reviews related literature. Section 3 introduces our model. Section 4 derives the optimal capacity investment portfolio. Section 5

analyzes the interactions between energy sources. Section 6 validates our main results by using real data through a case study. Section 7 considers several extensions of our model, including the effects of carbon tax. Section 8 concludes.

2 Literature Review

There is extensive literature in energy economics that studies capacity investment in conventional energy sources (see Crew et al. (1995) for a review of the early literature). With the advent of renewable energy, interest in this topic has increased in academia and practice because of the unique features of renewable energy: intermittency and negligible generation cost. It is not clear how the addition of this new energy source affects the investment portfolio. Lee et al. (2012) and Cochran et al. (2014) provide discussions on the interaction between renewable and flexible sources and argue that they can be complements. To determine optimal investment levels, researchers have used analytical models. For example, Garcia et al. (2012) and Kong et al. (2018) characterize capacity investment levels in renewable and conventional energy. However, they do not investigate the interactions between these energy sources.

Most papers that analytically study these interactions focus on two energy sources and conclude that renewable and conventional sources are substitutes. For example, Ambec and Crampes (2012) compare the optimal capacity portfolio in a centralized and a decentralized setting. Baranes et al. (2017) conduct a what-if analysis by varying the investment level of a conventional source to examine the corresponding optimal investment in renewable energy. Pinho et al. (2018) study the effects of renewable energy on electricity spot markets. Our paper is different from the aforementioned papers in that we jointly optimize the investment levels of three energy sources under a general stochastic demand and study their interactions in the optimal investment portfolio. Unlike these papers, we find that renewable and conventional sources can be complements.

To the best of our knowledge, Chao (2011) is the only paper that analytically characterizes the optimal investment portfolio and investigates the interactions between three energy sources: a wind farm, a combined-cycle natural gas turbine, and a regular natural gas turbine. Compared to the regular turbine, the combined-cycle turbine has higher investment and lower generation costs. Consequently, the combined-cycle turbine is similar to an inflexible source in our model and the

regular turbine is similar to a flexible source. In a simulation study, Chao (2011) observes that wind energy and the inflexible source (combined-cycle turbine) are substitutes, whereas wind energy and the flexible source (regular turbine) are complements. Our contribution is to analytically validate this insight.

Empiricists also explore this issue through data. Devlin et al. (2017) provide a summary of empirical papers and industry reports on the interaction between wind energy and natural gas from the perspectives of government policy, technical characteristics of power plants, and natural gas spot markets. Focusing on the electricity network of the U.K. and Ireland, Devlin et al. (2017) indicate that natural gas is crucial to the continued growth of renewable energy, suggesting that flexible natural gas-fired power plants and wind energy are complements. Marques et al. (2010) use the data from the E.U., where most natural gas-fired power plants are inflexible combined-cycle turbines. They find that a higher natural gas price leads to a lower investment in natural gas-fired power plants and a higher investment in renewable energy. On the other hand, Bushnell (2010) and Shrimali and Kniefel (2011) use data from the U.S., where most natural gas-fired plants are flexible open-cycle turbines. They find that natural gas-fired plants complement wind energy. Our analytical results reconcile these empirical findings.

In our model, we consider a utility firm whose objective is to minimize its total cost. Nevertheless, some papers consider a rate-of-return regulation under which a utility firm earns a guaranteed rate-of-return (e.g., 10%) over its cost. See, for example, Nezlobin et al. (2012). Our objective is not against the rate-of-return regulation, because a utility firm is more likely to satisfy the regulation if the cost of electricity generation is minimized. In fact, according to The Regulatory Assistance Project (2011, p.6), an important goal of the rate-of-return regulation is to minimize the cost of electricity generation.

Renewable energy has become an emerging topic in the operations management literature. Wu and Kapuscinski (2013) investigate how to cope with the intermittency of renewable sources. Al-Gwaiz et al. (2016) and Sunar and Birge (2018) characterize the supply function equilibrium in an electricity spot market. These papers do not endogenize capacity investment decisions in renewable and conventional sources. Hu et al. (2015) model capacity investments and show that as the granularity of the data on electricity demand and supply increases, more accurate investment decisions can be made. Aflaki and Netessine (2017) identify the critical role of intermittency in

determining the optimal capacity portfolio and effects of carbon tax policy. Kök et al. (2018) investigate the joint pricing and capacity investment problem for a utility firm and find that the renewable energy investment of the utility firm is higher under flat pricing compared to that under peak pricing. Although Hu et al. (2015), Aflaki and Netessine (2017), and Kök et al. (2018) study capacity investment in energy sources, their focus is different from ours. Their results indicate that renewable and conventional sources are substitutes. We refine this conclusion by modeling operational flexibility to show that renewable and flexible conventional sources can be complements.

Our definition of operational flexibility is similar to volume flexibility in the supply chain literature in which the production quantity can be altered depending on the realized demand. Van Mieghem and Dada (1999) consider postponing the production decision in a single source setting. Tomlin (2006) finds the importance of volume flexibility for a firm that can source either from an unreliable supplier or a reliable and flexible supplier. Volume flexibility is similar to quick response, where a firm can place additional orders after observing some initial information on demand (c.f., Fisher and Raman 1996). We complement this literature by jointly considering inflexible, flexible, and unreliable (renewable) sources to study the interactions between them.

Another flexibility type is process flexibility which is the ability to manufacture different products at the same facility (c.f., Fine and Freund 1990 and Jordan and Graves 1995). Several papers, including Van Mieghem (1998) and Goyal and Netessine (2011), study optimal capacity investments in two inflexible (dedicated) and one flexible source to meet the stochastic demand for two products. In this literature, the inflexible sources are complements with each other, and the flexible source is a substitute for both of them (Van Mieghem 1998). Different from these papers, which focus on demand-side uncertainty, we consider both demand- and supply-side uncertainties (due to the renewable source). We find that renewable and flexible sources are complements and the inflexible source is a substitute for both.

Finally, our paper relates to the dual sourcing literature in which a firm procures from two suppliers: the first supplier features a long lead time and a low procurement cost, whereas the second has a short lead time and a high cost. In this literature, Sting and Huchzermeier (2012) is closest to our paper. The authors consider a manufacturer who invests capacity in a responsive, onshore facility and also replenishes from an offshore supplier who is unreliable but less expensive. After demand and supply uncertainties are realized, the manufacturer orders from its responsive capacity

to satisfy the demand. Sting and Huchzermeier (2012) characterize the optimal production policy and show that the service level is determined by the critical fractile of the responsive capacity. We extend these results by considering three sources: the two reliable sources, that is, the flexible and the inflexible sources, can be viewed as an onshore and an (reliable) offshore supplier respectively, the intermittent renewable source can be viewed as an (unreliable) onshore supplier. Different from their findings, our results suggest that not all sources are substitutes, although any two source combinations (with the investment level of the third source fixed) of our model gives the same result as Sting and Huchzermeier (2012).

3 Model

To facilitate the formulation of our model, we first describe how a monopolist utility firm makes its capacity investment decision in practice. The typical process starts from forecasting the electricity demand and the intermittency of renewable energy supply in a geographical region. The uncertainties of demand and intermittency are correlated because of the common effect of weather conditions. In addition, demand and intermittency are serially correlated (as time series) and non-stationary over time: the demand changes throughout a day (EIA 2011c) and wind energy exhibits seasonal fluctuations (EIA 2015). Using the demand and supply forecasts as an input, the utility firm makes a strategic decision on its investment level in inflexible, renewable, and flexible energy sources. The investment level of a source is the maximum output that the utility firm can dispatch from that source. In daily operations, the utility firm's objective is to match the random demand with the electricity supply for each operating period, which is often set to be five minutes. The utility firm uses the five-minute-ahead forecasts of demand and supply as inputs and decides how much electricity to generate from the renewable and flexible sources in each operating period.[1] The inflexible source, on the other hand, is dispatched at a constant level. This is because a utility firm cannot frequently change the output of an inflexible source due to technical reasons (c.f., Shively and Ferrare 2008, p. 39, Denholm et al. 2010, and DOE 2011).

An important input for the dispatch decision is the short-term demand and supply forecasts,

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[1]In general, renewable energy is not curtailed. That is, the entire capacity of the renewable source is dispatched because its generation cost is negligible.

which are quite accurate.[2] In Figure 1, we plot the Mean Absolute Percentage Error (MAPE) for demand and intermittency forecasts in 2014 for the Southwest Power Pool (SPP), the network of utility firms in the southwest U.S. Each circle represents one of the 288 operating periods (i.e., five-minute intervals) during a day. For each period, we plot the average error over a year for day-ahead and five-minute-ahead forecasts in the vertical and horizontal axes, respectively. All circles remain well above the 45 degree line, indicating that the forecasts made five-minutes ahead are much more accurate compared to the forecasts made a day ahead. Thus, a utility firm has relatively accurate forecasts of demand and supply before determining the dispatch quantities.

(a) Demand Forecasts   (b) Intermittency Forecasts
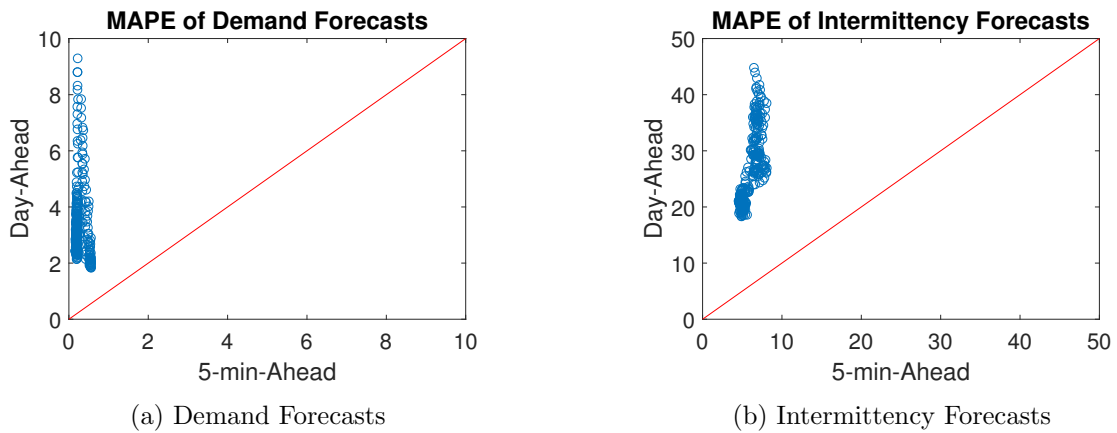
Figure 1: Forecast Errors in the Southwest Power Pool, 2014

The costs involved in the above process are the investment costs and the generation (variable) costs of electricity. Specifically, the generation cost of the renewable source is negligible, and the generation cost of inflexible sources, such as nuclear or coal-fired power plants, is usually smaller than that of flexible sources, such as natural gas (EIA 2017). In some rare occasions, blackouts occur if the demand cannot be fulfilled by the dispatched supply. Blackouts are costly because a utility firm usually needs to purchase electricity from external sources in order to avoid fines imposed by government regulations. The objective of the utility firm is to minimize the total cost consisting of the investment costs, generation costs, and penalty costs due to potential blackouts. We refer to two latter costs as generation-related costs.

We formulate our problem as a stochastic program with recourse based on the above practice. We consider N operating periods in the planning horizon. That is, on the operational level, we

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[2]In practice, the dispatch decision may also include a day-ahead planning phase.

consider multi-period dispatch decisions. On the strategic level, we only focus on one-time capacity investments.[3] The problem consists of two stages: the first stage is related to the initial capacity investment decision, and the second stage is related to the dispatch decision to match the demand with the supply. Let the variable generation cost (in dollars per unit capacity for a period) of the inflexible and flexible sources be c_I and c_F, respectively. We normalize the variable cost of the renewable source to zero ($c_R = 0$). The demand and intermittency follow a nonstationary and serially correlated stochastic process, such as a vector autoregressive model in addition to trend and seasonality (see Appendix A). Let the joint probability density function of the demand and supply in period n be $f_{(\Xi_n, \Theta_n)}(\cdot, \cdot)$, where Ξ_n is a bounded, nonnegative random variable that represents the five-minute-ahead demand forecast and, Θ_n is a random variable with a support of [0,1] representing the intermittency forecast. That is, renewable energy investment of k_R results in an electricity output of $\Theta_n k_R$ in period n. Here, Ξ_n and Θ_n are correlated, and the joint distribution $f_{(\Xi_n, \Theta_n)}(\cdot, \cdot)$ represents the marginal distribution (with respect to time) of the demand and intermittency process. The sequence of events is illustrated in Figure 2.
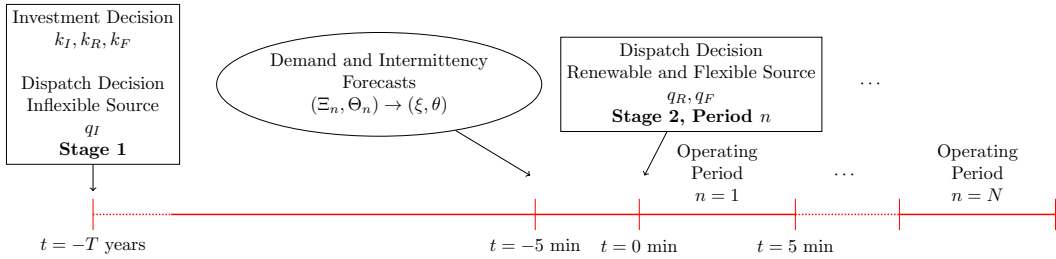

Figure 2: Sequence of Events

We formulate the problem backwards. Let q_I, q_R, and q_F denote the dispatch levels of the inflexible, renewable, and flexible sources, respectively. Similarly, k_i denotes the investment level in source $i \in \{I, R, F\}$. Any unmet demand results in an undersupply penalty cost, with rate r, proportional to the amount of electricity demand that cannot be satisfied by the dispatched electricity from the three sources. This linear penalty cost is consistent with the literature (c.f., Crew et al. 1995), and our model can be generalized by considering an oversupply penalty (see Section 7.2). The second stage problem of the utility firm is to minimize the sum of generation-

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[3]We assume that the utility firm forms a capacity portfolio without any existing investment. Our results can be extended to the case in which the existing generation capacity has the same generation cost as the new capacity.

related costs for each period n after observing demand and intermittency forecasts ξ and θ:

$$\tilde{C}(q_I, k_R, k_F, \xi, \theta) = c_I q_I + \begin{cases} \min_{q_R, q_F \geq 0} & c_F q_F + r(\xi - q_I - q_R - q_F)^+ \\ \text{subject to} & q_R \leq \theta k_R \\ & q_F \leq k_F \end{cases}, \qquad (1)$$

where $(x)^+ = \max\{x, 0\}$.

In the above formulation, the decision variables are the dispatch levels of the renewable and flexible sources, whereas the dispatch level of the inflexible source q_I is given as a state variable. This is because the inflexible source is dispatched at a constant level, which cannot be adjusted in each period. Thus, we consider q_I as a long-term decision and optimize it in the subsequent first stage problem. This formulation implicitly assumes that the inflexible source will be dispatched earlier than the other sources, which is consistent with the current practice.[4] Recall that ξ and θ in (1) are the five-minute-ahead forecasts of the demand and the intermittency, respectively. As explained above, these forecasts are quite accurate. Hence, as in Wu and Kapuscinski (2013), we take ξ and θ as the realizations of the demand and the supply uncertainty, respectively.

In the first stage problem, the utility firm determines its nonnegative capacity investment levels along with the dispatch decision of the inflexible source so as to minimize its expected total cost:

$$\min_{\boldsymbol{k} \in \mathbb{R}^3_+} \bar{\Pi}(\boldsymbol{k}) = \alpha_I k_I + \alpha_R k_R + \alpha_F k_F + \min_{0 \leq q_I \leq k_I} E\left[\sum_{n=1}^{N} \tilde{C}(q_I, k_R, k_F, \Xi_n, \Theta_n)\right], \qquad (2)$$

where $\boldsymbol{k} = (k_I, k_R, k_F)$, α_i is the unit investment cost in source $i \in \{I, R, F\}$, $E[\cdot]$ denotes the expectation operator, N is the number of operating periods, and $\tilde{C}(q_I, k_R, k_F, \Xi_n, \Theta_n)$ is the solution of the second stage problem given in (1). The expectation is taken with respect to the joint distribution of demand and supply (Ξ_n, Θ_n) in period n from the perspective of period 0, that is, the planning stage for the utility firm. Here, in addition to the capacity investment levels, the utility firm determines the dispatch level of the inflexible source.

This stylized model makes simplifying assumptions to ensure tractability. First, as in Al-Gwaiz

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[4]In fact, it is also optimal to first dispatch the renewable source under this formulation. That is, it is also optimal to set $q_R = \theta k_R$ in all periods because $c_R = 0$, and there is no explicit oversupply penalty. Nevertheless, we explicitly consider q_R to ensure consistency with the second stage problem of the spot market setting given in (13)–(16). In the presence of a spot market, it might not be optimal to dispatch all renewable capacity as we explain in Section 7.1. Finally, in Section 7.2, we also consider an explicit oversupply penalty.

et al. (2016), we suppose that between consecutive periods, the output of a conventional source either cannot be changed at all or can be changed instantaneously without any constraint. This is an approximation of the practice where each power plant has a different level of flexibility based on its generation characteristics. These characteristics are considered in the case study in Section 6, and we find that our conclusions continue to hold. Second, we consider a monopolist utility firm that does not have access to an electricity spot market. That is, the firm is responsible for matching supply and demand by using its own generation sources. This is not uncommon in practice because approximately half of U.S. utility firms operate as a monopoly (FERC 2015b). Nevertheless, we consider an electricity spot market in Section 7.1. Finally, although we do not need any assumptions on the joint distribution of the demand and supply uncertainty in characterizing the optimal capacity portfolio, we require certain sufficient conditions to hold in analyzing the interactions between energy sources. We present and discuss these conditions in Section 5 as Assumption 1.

In the remainder of the paper, we use the terms "increasing," "decreasing," and "convex" in the weak sense. We denote the gradient operator as ∇. Finally, "$X|\cdot$" denotes the conditional probability. All proofs and parameter values for numerical studies are given in the Appendix.

4   Optimal Capacity Investments

In this section, we characterize the optimal capacity investments of a utility firm. We first simplify the problem given in (2) by showing that at optimality, the dispatch level of the inflexible source is always equal to its capacity investment level, i.e., $q_I = k_I$. The intuition is that the firm should always dispatch all of its inflexible capacity k_I at every period because the firm can otherwise achieve a strictly lower cost by decreasing k_I.

Lemma 1. *Consider the investment problem given in (2). It is optimal to set $q_I = k_I$.*

Lemma 1 is consistent with the practice because the utilization of nuclear power plants in the U.S. is close to 90% (EIA 2017), and these plants operate continuously, except for maintenance (EIA 2011b). By using Lemma 1, we substitute k_I for q_I in the second stage dispatch problem

given in (1):

$$C\left(\boldsymbol{k}, \xi, \theta\right) = \min_{q_R, q_F \geq 0} \quad c_F q_F + r\left(\xi - k_I - q_R - q_F\right)^+ \tag{3}$$

$$\text{subject to} \quad q_R \leq \theta k_R \tag{4}$$

$$q_F \leq k_F. \tag{5}$$

Similarly, under Lemma 1, the capacity investment problem in the first stage becomes

$$\min_{\boldsymbol{k} \in \mathbb{R}_+^3} \bar{\Pi}\left(\boldsymbol{k}\right) = \left(\alpha_I + c_I N\right) k_I + \alpha_R k_R + \alpha_F k_F + E\left[\sum_{n=1}^N C\left(\boldsymbol{k}, \Xi_n, \Theta_n\right)\right], \tag{6}$$

where we charge the generation cost of the inflexible source to its entire capacity for each of the N periods. In the remainder of the paper, we focus on these simplified formulations of the first and second stage problems.

We next characterize the optimal capacity investments by backward induction, i.e., by first solving the second stage problem given in (3)–(5). Let $q_i^*\left(\boldsymbol{k}, \xi, \theta\right)$ be the optimal dispatch level of energy source $i \in \{R, F\}$ given an investment vector $\boldsymbol{k}$, demand forecast ξ, and intermittency forecast θ. The optimal dispatch policy for renewable and flexible sources is shown below.

Lemma 2. *Consider the dispatch problem given in (3)–(5). The optimal dispatch policy is to set* $q_R^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(\theta k_R, \xi - k_I\right)^+$ *and* $q_F^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(k_F, \xi - k_I - \theta k_R\right)^+.$

Lemma 2 shows that the utility firm first dispatches its renewable source up to its available capacity θk_R if demand forecast ξ exceeds the inflexible source capacity k_I in a period. Then, the flexible source is dispatched for the remaining demand. This is because the renewable source incurs a negligible generation cost compared to the flexible source. Lemmas 1 and 2 conclude the optimal dispatch policy: in every period, all of the inflexible capacity is dispatched, followed by the renewable source, and then by the flexible source.

We next use this optimal dispatch policy to characterize the optimal capacity portfolio. Our analysis involves constructing the dual of the dispatch problem in (3)–(5) such that $\lambda_i^*\left(\boldsymbol{k}, \xi, \theta\right)$ denotes the optimal dual variable associated with the capacity constraint related to source $i \in \{I, R, F\}$. We present this dual problem in the proof of Proposition 1, where each dual variable represents the shadow price of the associated capacity constraint.

Lemma 2 is obtained by solving the dispatch problem based on the realizations of demand and

Table 1: Shadow Prices of Capacity Constraints for Demand and Intermittency Space Partitions

Partition for $(\xi, \theta) \in \mathbb{R}_+ \times [0,1]$	$\lambda_I^* (\boldsymbol{k}, \xi, \theta)$	$\lambda_R^* (\boldsymbol{k}, \xi, \theta)$	$\lambda_F^* (\boldsymbol{k}, \xi, \theta)$
$\Omega_1 (\boldsymbol{k}) = \{(\xi, \theta) \| \xi \leq k_I + \theta k_R\}$	0	0	0
$\Omega_2 (\boldsymbol{k}) = \{(\xi, \theta) \| k_I + \theta k_R < \xi \leq k_I + \theta k_R + k_F\}$	c_F	θc_F	0
$\Omega_3 (\boldsymbol{k}) = \{(\xi, \theta) \| k_I + \theta k_R + k_F < \xi\}$	r	θr	$r - c_F$

supply uncertainty. There are three regions of the uncertainty space in each of which the optimal dispatch decision as well as the dual variables have the same structure. We present these regions in Table 1. For example, in Ω_1, ξ and θ are such that $\xi \leq k_I + \theta k_R$, i.e., the demand is less than the sum of the inflexible and available renewable capacity. Then, it is optimal to set $q_R = (\xi - k_I)^+$ and $q_F = 0$, as also indicated by Lemma 2. Furthermore, in this case, no capacity constraint is binding so that all dual variables are zero. The uncertainty regions are identical across all N periods, but the probability that a pair of (ξ, θ) falls into a specific region in each period depends on the (non-identical) joint distribution of (Ξ_n, Θ_n). In addition, because $\bar{\Pi}(\boldsymbol{k})$ is convex, the Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for the investment problem given in (6). Moreover, we can show that $\nabla_{\boldsymbol{k}} E[C(\boldsymbol{k}, \Xi_n, \Theta_n)] = -E[\boldsymbol{\lambda}(\boldsymbol{k}, \Xi_n, \Theta_n)]$ for all n. That is, the derivative and the expected value can be interchanged, where the expected value of the dual variables can be easily computed by using Table 1. With these observations, in Proposition 1, we present the KKT conditions of the investment problem given in (6), where $\boldsymbol{v}$ is the vector of Lagrange multipliers of the nonnegativity constraints. Here, $P^n(\Omega_j)$ is the probability that for (Ξ_n, Θ_n), ξ and θ are in Ω_j, where Ω_j is defined in Table 1 such that $P^n (\Omega_1 \cup \Omega_2 \cup \Omega_3) = 1$.

Proposition 1. *Consider the problem given in (3)–(6). An investment vector $\boldsymbol{k}^* \in \mathbb{R}_+^3$ is optimal if and only if there exists a $\boldsymbol{v} \in \mathbb{R}_+^3$ such that*

$$\sum_{n=1}^{N} \left(E \left[\left. \begin{matrix} c_F \\ \Theta_n c_F \\ 0 \end{matrix} \right| \Omega_2 (\boldsymbol{k}^*) \right] P^n (\Omega_2 (\boldsymbol{k}^*)) + E \left[\left. \begin{matrix} r \\ \Theta_n r \\ r - c_F \end{matrix} \right| \Omega_3 (\boldsymbol{k}^*) \right] P^n (\Omega_3 (\boldsymbol{k}^*)) \right) = \left(\begin{matrix} \alpha_I + c_I N - v_I \\ \alpha_R - v_R \\ \alpha_F - v_F \end{matrix} \right) .$$

$$\tag{7}$$

$$\forall i \in \{I, R, F\} : k_i v_i = 0. \tag{8}$$

Equation (7) is obtained by taking the partial derivative of the Lagrangian function with respect to k_I, k_R, and k_F, respectively. The expectations in (7) are taken with respect to the joint distri-

bution of the demand and intermittency uncertainties. Based on these KKT conditions, there are a total of eight cases that we should consider in order to find the optimal investment levels. These eight cases form four investment strategies: (i) no investments (i.e., $\boldsymbol{k}^* = 0$), (ii) single sourcing (three cases, e.g., $k_I^* > 0$ and $k_R^* = k_F^* = 0$), (iii) dual sourcing (three cases, e.g., $k_I^*, k_R^* > 0$ and $k_F^* = 0$), and (iv) triple sourcing (i.e., $\boldsymbol{k}^* > 0$). No investments strategy is optimal if $rN < \alpha_i + c_i N$ for $i \in \{I, F\}$, and $r \sum_{n=1}^{N} E[\Theta_n] < \alpha_R$, i.e., if the investment and generation costs are higher than the penalty cost. Unfortunately, we are not able to analytically characterize the range of cost parameters that ensures the optimality of the rest of the investment strategies due to the non-stationarity in demand and supply uncertainty. Nevertheless, based on the estimates of the cost parameters and the data of Texas, we observe that the triple sourcing strategy is optimal. This is consistent with the practice that utility firms jointly invest in inflexible, renewable, and flexible sources (FERC 2015a). Motivated by these facts, in the subsequent discussion, we focus on the triple sourcing strategy, as this is the most interesting and relevant case. We also investigate the other strategies in Section 7.5.

Proposition 1 provides a method to find the optimal investment levels for the triple sourcing investment strategy. The idea is to solve three newsvendor problems simultaneously with $\boldsymbol{v} = 0$ in (7), each corresponding to one energy source. Specifically, for the inflexible source, the underage cost includes the expectation of two events associated with the demand exceeding the capacity of this source. In the first case, the capacity of the flexible source is sufficient to meet the remaining demand. In the second, the total demand may exceed the entire capacity, and a penalty cost r is incurred in addition to the generation cost of the flexible source. Hence, the underage cost for the inflexible source is the probability weighted sum of these two costs. The overage cost for the inflexible source, on the other hand, is the investment and the generation cost. Note that we include the generation cost of the inflexible source in the overage cost because the entire capacity of this source is dispatched at every period even if its capacity exceeds the demand.

For the renewable source, the underage cost is similar to the inflexible source. However, supply uncertainty Θ_n is also considered while computing the expectation. The overage cost only includes the investment cost but not the variable generation cost for two reasons. First, we assume that the variable cost is zero for the renewable source. Second, even in the absence of this assumption, the utility firm would not dispatch the renewable source if its capacity exceeds the demand, so the

variable generation cost should not be included in the overage cost.

For the flexible source, the underage cost only involves the event of demand exceeding the total capacity. In this case, the penalty cost is incurred and the underage cost is given as $(r - \alpha_F - c_F)$. Note that we deduct the investment and generation cost from the penalty cost, i.e., as in the classical newsvendor model, we consider the net underage cost. The overage cost for the flexible source is only the capacity cost, α_F.

In summary, the optimality condition suggests that there is a pair of underage cost and overage cost that determines the optimal investment level for each energy source. The utility firm balances the underage and overage costs of inflexible, renewable, and flexible sources for demand and supply realizations, as shown in Figure 3, where we assume that $k_F > k_R$ for illustration purposes. The thick line in the figure represents the maximum demand that the firm is able to serve. By adjusting its investments, the utility firm determines the probability of each region so that the underage cost is balanced with the overage cost for each energy source.
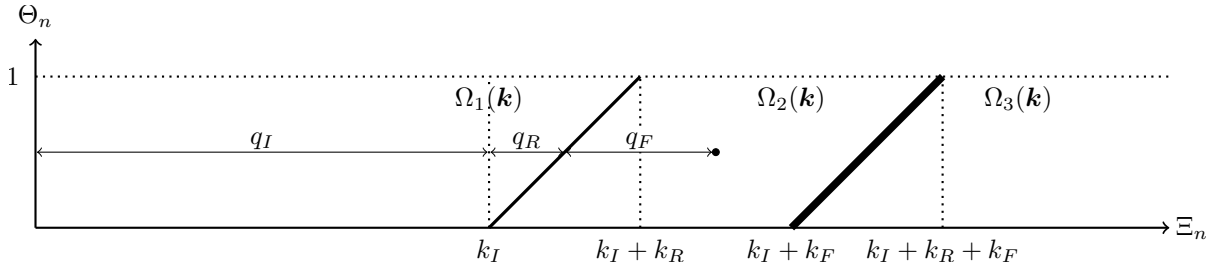

Figure 3: Partitions of Demand and Intermittency Space

Next, we consider the relationship between the investments and the reliability of the electricity grid. In the energy economics literature, reliability is defined based on the loss-of-load probability (LOLP), i.e., the probability that the demand exceeds the supply of electricity (Telson 1975). This definition is similar to the concept of service level in the supply chain management literature. We note that LOLP is not the same as the probability of a blackout because the utility firm may procure electricity from an external source to avoid a blackout. Let ρ^* denote the LOLP corresponding to the optimal investment levels:

$$\rho^* = \sum_{n=1}^{N} P^n(\Omega_3(\boldsymbol{k}^*)), \tag{9}$$

where $\Omega_3(\boldsymbol{k}^*)$ is the demand and intermittency region in which the demand exceeds the available supply.

Corollary 1. $\rho^* = \alpha_F/(r - c_F)$, *where ρ^* is defined in* (9).

In the triple source strategy (i.e., $\boldsymbol{k}^* > 0$), Corollary 1 immediately follows from the third dimension of the optimality condition (i.e., with respect to k_F) given in (7) in Proposition 1. It suggests that the reliability of the electricity grid is only affected by the penalty cost rate r and the cost parameters of the flexible source in the triple source strategy. That is, the newsvendor critical fractile of the flexible source determines the service level. Intuitively, the flexible source is the last option for the utility firm to satisfy the demand, and the firm finds the optimal investment level in this source by comparing the penalty cost of not satisfying the demand against the investment cost. This result is an extension of similar observations made in the energy economics (c.f., Chao 1983) and dual sourcing literature (c.f., Sting and Huchzermeier 2012) to our setting.

Corollary 1 suggests an important policy insight. Because subsidies for the renewable or inflexible source do not affect r, c_F, or α_F, these subsidies do not change the reliability of the grid. This result provides a different perspective from the claims that renewable energy subsidies undermine reliability, and nuclear subsidies enhance reliability (c.f., Gronewold 2011, Garman and Thernstrom 2013, Karnitschnig 2014, Fisher 2015, and Smith 2015). This is because our model optimizes investments in all energy sources simultaneously and can identify the effect of subsidies on the entire capacity portfolio rather than consider the effect on a single source.

5   Interaction Between Energy Sources

In this section, we investigate how providing a subsidy for one energy source affects the investment level of other energy sources. Two consumption goods are substitutes if a decrease in the price of one good leads to a lower level of consumption in the other (Singh and Vives 1984). From the utility firm's perspective, energy sources are consumption goods, and their price is the investment cost. Hence, we define two energy sources as *substitutes* if a decrease in one's investment cost leads to a decrease in the other's investment level. That is, sources i and j are substitutes if a decrease in α_i leads to a decrease in k_j^* (i.e., $dk_j^*/d\alpha_i > 0$) and vice-versa (i.e., $dk_i^*/d\alpha_j > 0$). Analogously, we define two sources as *complements* if a decrease in one's investment cost leads to an increase in the other's investment. We refer to the decrease in investment cost as an investment subsidy. In practice, this decrease is not limited to the subsidies provided by the government but

can also represent a technological improvement that reduces the investment cost. For example, a new technology has reduced investment cost for coal-fired power plants (Duke Energy 2015b), which can be considered as a decrease in α_I. We first present a preliminary result before identifying the interactions between energy sources (i.e., how a subsidy for one source affects investment in others).

Proposition 2. *For $i, j \in \{I, R, F\}$, (i) $\frac{dk_i^*}{d\alpha_i} \leq 0$, (ii) $\frac{dk_i^*}{d\alpha_j} = \frac{dk_j^*}{d\alpha_i}$.*

Proposition 2(i) shows that providing a subsidy for an energy source leads to a higher investment level in that source. Intuitively, the subsidy leads to a lower investment cost; in response, the utility firm increases its investment. Part (ii) shows that the cross effect of a subsidy is symmetric: the change in the investment level for source i in response to a change in the investment cost of source j is equivalent to that for source j in response to a change in the investment cost of source i.

To examine the interactions between energy sources, we make the following assumption. Define

$$g\left(\xi, \theta\right) = \sum_{n=1}^{N} f_{(\Xi_n, \Theta_n)}\left(\xi, \theta\right) \tag{10}$$

as the sum of the joint density function of demand and intermittency distributions over N periods.

Assumption 1. *(i) $g\left(\xi, \theta\right)$ defined in (10) is log-concave in ξ for any θ. (ii) $\frac{g(\xi, \theta_2)}{g(\xi, \theta_1)}$ is decreasing in ξ for any $\theta_2 \geq \theta_1$.*

Below, we discuss the implications of this assumption for wind and solar energy separately because they have different generation patterns.

Wind Energy

To test the practicality of Assumption 1, we evaluate the $g(\xi, \theta)$ function by using the realized electricity demand and wind energy intermittency data between the years of 2016 and 2018 in the Southwest Power Pool (SPP). As we explain in detail in Appendix A, we fit a serially correlated and nonstationary process to the data and estimate its parameters. The process consists of trend, seasonality, and a noise component that follows a vector autoregressive (VAR) model of order 1. By using the estimates, we characterize $f_{(\Xi_n, \Theta_n)}\left(\xi, \theta\right)$ for all n as a nonstationary bivariate normal distribution (Huang and Schneider 2011) and evaluate the $g(\xi, \theta)$ function.

In Figure 4a, we plot $\log g(\xi, \theta)$ for four different θ values and observe that it is concave, consistent with Assumption 1(i). This condition holds if each density function is log-concave and sufficiently similar (i.e., stationary) because log-concavity is preserved under multiplication by a constant. We note that many commonly used distributions, including Normal, Logistic, and Extreme Value, have log-concave density functions (Bagnoli and Bergstrom 2005). Assumption 1(ii) is related to the decreasing likelihood ratio property. Intuitively, this condition is satisfied if the electricity demand and the intermittency are negatively correlated. This is the case for wind energy. As shown in Figure 4b, $\frac{g(\xi, \theta_2)}{g(\xi, \theta_1)}$ is decreasing in ξ in three cases where $\theta_2 \geq \theta_1$, consistent with Assumption 1(ii). Hence, Assumption 1 is satisfied by the real electricity demand and wind energy supply data of the SPP. We use this assumption as a sufficient condition in presenting our main results below.
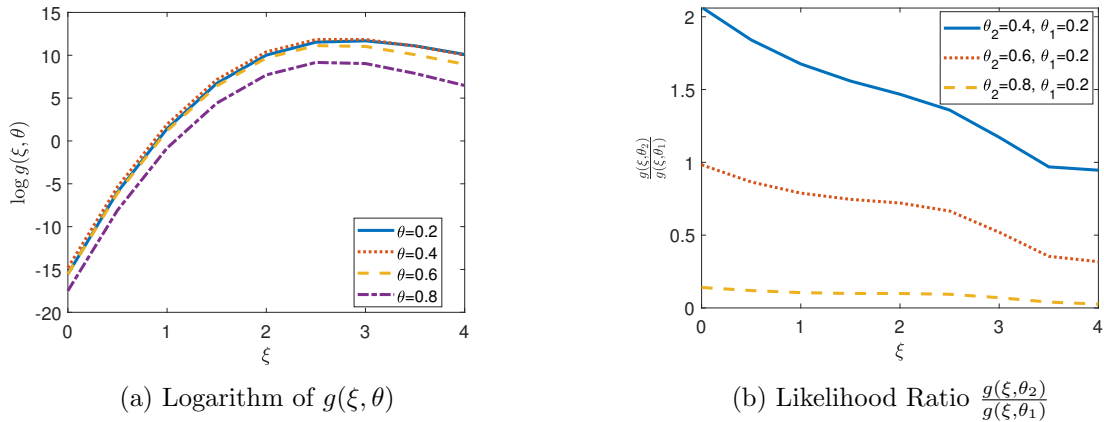

(a) Logarithm of $g(\xi, \theta)$

(b) Likelihood Ratio $\frac{g(\xi, \theta_2)}{g(\xi, \theta_1)}$

Figure 4: Practicality of Assumption 1 in the Southwest Power Pool

Proposition 3. *(i) The inflexible and renewable sources are substitutes. If Assumption 1 holds, then (ii) the inflexible and flexible sources are substitutes, and (iii) the renewable and flexible sources are complements.*

Proposition 3(i) and (ii) indicate that a subsidy for the inflexible source leads to a lower investment level in the renewable and flexible sources. However, Proposition 3(iii) shows that a subsidy for the flexible source leads to a higher investment in the renewable source. This is a new insight compared to the dual sourcing literature, which suggests that the two sources considered in a dual sourcing case are substitutes. We explain the intuition behind these results based on the subsidy

for the flexible source. By considering other subsidies, we can obtain similar insights because the cross effects of the subsidies are equivalent by Proposition 2.

The flexible source subsidy leads to an increase in the investment level of the flexible source, which alleviates the intermittency problem of wind energy. This *intermittency alleviation effect*, in turn, encourages the utility firm to invest more in wind energy in order to take advantage of its negligible generation cost. Consequently, the increased investment in both wind energy and the flexible source lowers the investment level of the inflexible source. We illustrate these results in Figure 5a based on real data from the state of Texas (see Section 6). As shown in the figure, when the flexible source subsidy increases (i.e., α_F decreases), the investment levels of both renewable and flexible sources increase, but the investment level of the inflexible source decreases.

We next discuss the role of Assumption 1 on the substitution and complementarity effects. First, the log-concavity condition in Assumption 1(i) intuitively means that the demand distribution has sub-exponential tails (i.e., light-tailed) for any given level of intermittency (An 1998). If this condition is violated, the complementarity effect between the flexible and the renewable source may not hold. This is because if the demand distribution is heavy-tailed, it becomes likely that the demand takes arbitrarily high values, which can only be satisfied by the flexible source (because the renewable source is intermittent). Hence, in response to a subsidy for the flexible source, the utility firm may increase the investment in the flexible source significantly and reduce the investment of the intermittent renewable source. Second, recall that Assumption 1(ii) requires a negative correlation between the electricity demand and the renewable energy supply. The substitution and complementarity effects are stronger if the correlation is negative, as in the case of wind energy. To see the intuition, consider a specific five-minute interval to avoid the complication of the nonstationarity in demand and intermittency. In this interval, if the demand level is low, the optimal dispatch policy suggests that the demand is mostly satisfied by the inflexible source. When the demand level is low at nighttime, due to the negative correlation, the wind output tends to be higher, reducing the need for the inflexible source. Hence, the substitution effect is stronger between wind energy and the inflexible source. Under a negative correlation, the complementarity effect between wind energy and the flexible source is also stronger. This is because the flexible source is used more when the demand level is high. When the demand is high during the daytime, wind output tends to be low, increasing the need for the flexible source and strengthening the

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complementarity effect.

Solar Energy

Proposition 3 requires Assumption 1 as a sufficient condition. Recall that Assumption 1(ii) stipulates that the electricity demand and the renewable energy supply are negatively correlated.[5] For solar energy, the correlation between the supply and demand may be positive, as higher solar output correlates with warmer weather, which may increase electricity usage. This is, in fact, the case in the state of Texas, where the correlation coefficient is 0.36. As explained above, the positive correlation weakens the complementarity effect between flexible and renewable energy sources. Consequently, we are unable to establish analytical results for solar energy.

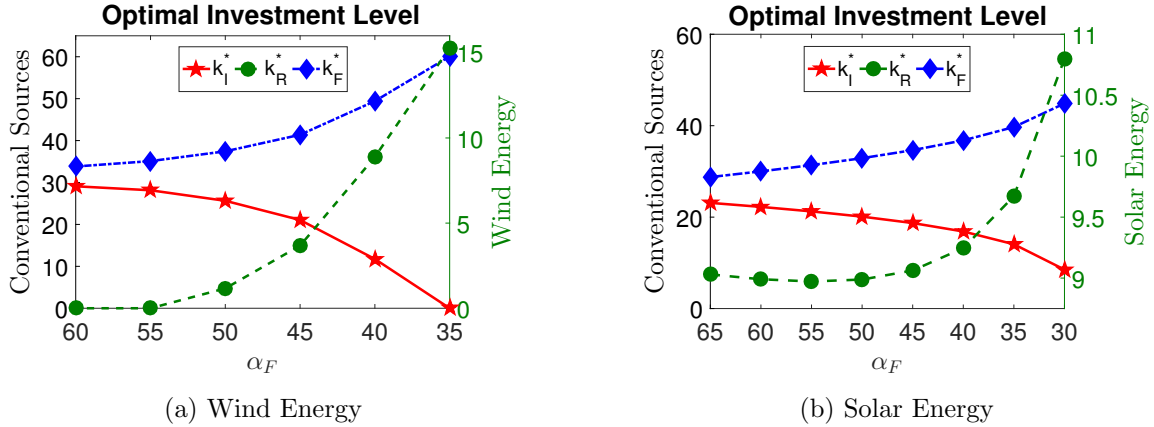(a) Wind Energy   (b) Solar Energy

Figure 5: Effect of a Subsidy for the Flexible Source

To analyze whether the results of Proposition 3 hold for solar energy, we present another numerical study in Figure 5b based on real data from Texas, where solar energy and demand are positively correlated. Although solar energy and the flexible source remain to be complements for most problem parameters, they are no longer complements if the subsidy level for the flexible source is low (i.e., α_F is high). This interesting result illustrates the complexity of identifying the interaction between energy sources. Specifically, the positive correlation between the demand and solar energy weakens the complementarity between solar energy and the flexible source. In fact,

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[5]Assumption 1(ii) may be satisfied by the solar energy investment of a utility firm if the utility firm operates in a region with a significant penetration of household solar panels. In these regions, such as California, the utility firm satisfies the net customer demand, i.e., demand minus the generation from household panels. The net demand and the solar energy investment of the utility firm may exhibit a negative correlation; hence, all of the above results given for wind energy continue to apply. That is, the solar energy investment of the utility firm is complemented by the flexible source.

the complementarity effect is reversed if α_F is between 55 and 65 \$/kW in Figure 5b. This is because the solar energy output tends to be higher in the daytime when the demand level is also high, and the flexible source is mostly used to satisfy the demand. As a result, as the investment in the flexible source increases (due to the subsidy), the need for solar energy decreases, making the two sources substitutes. We further observe that as the subsidy level increases (i.e., α_F decreases), the investments in both the flexible source and solar energy increase, indicating that these sources become complements. That is, the intermittency alleviation effect outweighs the effect of the positive correlation if the subsidy level in the flexible source is sufficiently high.

6   Case Study: Texas Data

We next validate our main insights by using real electricity generation and demand data from the state of Texas. In our analytical model, given in (3)–(6), we assume that between consecutive periods, the output of a flexible source can be changed instantaneously and the output of an inflexible source cannot be changed at all. However, in practice, operational flexibility depends on plant-level characteristics. For example, there is a limit on how fast the output of a flexible source can be ramped up. In this case study, by considering these characteristics, we validate our results on the complementarity and substitution effects between energy sources.

Table 2: Sample Plant Characteristics for Texas, 2010

Plant Name	Plant Type	Minimum Output (MW)	Minimum Downtime (hr)	Startup Cost (\$)	Ramp Up Limit (%/min)
South Texas Project	Nuclear	812	168	15,000	1
Morgan Creek	Open-Cycle Natural Gas	122	0.5	1,203	10

Table 2 illustrates generation characteristics that determine operational flexibility for a representative nuclear and natural gas power plant in Texas (Cohen 2012). Here, the minimum output, minimum downtime, and startup cost (i.e., the cost of extra fuel to start the plant after it has been shut down) are all greater for the nuclear power plant than those for the natural gas plant. Furthermore, a utility firm can increase the output of the natural gas plant by 10% of its capacity every minute, but the nuclear plant can only be ramped at a rate of 1% of its capacity. In practice,

a utility firm considers these salient features and determines the least costly way of satisfying the electricity demand with its available set of generators. In doing so, the firm uses a *unit commitment and dispatch model* (UCDM), a mixed integer program that minimizes the generation cost subject to electricity system constraints, such as capacity limits, ramp up/down constraints, and minimum up/down times. We use Cohen (2012)'s dispatch model that mimics the operations in Texas electricity system to determine how providing a subsidy for one source affects the investment in other sources.

Next, we describe the data used in the UCDM. As an input, the UCDM uses the demand data and generation characteristics of available power plants. We use the observed 15-minute demand data from the state of Texas in 2010. For the generation mix (available set of power plants), we use the same data sources as in Kök et al. (2018). That is, we utilize the rich dataset given in Cohen (2012) that reports various generation characteristics, including those related to the operational flexibility of the 144 conventional power plants in Texas. Furthermore, for wind energy, we use the 15-minute output data which is also provided by Cohen (2012). For the solar energy output, because solar capacity was negligible in Texas in 2010, we rely on the simulation study of Kök et al. (2018).

We now turn to our analysis to identify the interaction between energy sources. We first determine the optimal capacity investment in inflexible, renewable, and flexible sources in Texas electricity system for current estimates of investment costs. Then, to investigate how optimal investment levels change, we decrease the investment cost of each source sequentially, which corresponds to providing a subsidy for each source. Specifically, the utility firm minimizes its generation and investment cost by determining its investment level in the three energy sources:

$$\min_{k_I, k_R, k_F} \bar{\Pi}(k_I, k_R, k_F) = \alpha_I k_I + \alpha_R k_R + \alpha_F k_F + G(k_I, k_R, k_F), \tag{11}$$

where the first three terms are the investment costs, and $G(k_I, k_R, k_F)$ is the output of the UCDM given inflexible, renewable, and flexible source investments of k_I, k_R, and k_F, respectively. In essence, we use the UCDM instead of the second stage problem of the analytical model in (3)–(6).

To determine the optimal investment levels, we next evaluate $G(k_I, k_R, k_F)$ at various k_I, k_R, and k_F levels, considering wind or solar energy as the renewable source. Each evaluation takes 1.2 CPU hours on average; hence, we only consider a limited set of investment levels. In particular, we take the current level of investment in energy sources as a basis and evaluate $G(k_I, k_R, k_F)$ at current

investment levels, as well as when additional investments are made. We consider nuclear energy as the inflexible source and natural gas-fired steam boilers as the flexible source. We allow additional investments of $\{0, 1000, 3000, 5000\}$ MW for both conventional sources. For the renewable source, we consider additional investments of $\{0, 5000, 10000, 15000, 20000\}$ MW. We consider a maximum investment level of 20,000 MW for the renewable source to ensure that the expected output from the renewable and conventional sources is similar. For example, wind source is intermittent with a capacity factor of approximately 0.3, meaning that the effective capacity is $6,000 = (20,000 \times 0.3)$ MW for wind energy.

In summary, we enumerate $G(k_I, k_R, k_F)$ for 160 cases: 4 levels of nuclear investments by 4 levels of natural gas investments by 5 levels of renewable investments by 2 different renewable sources. Among these cases, under current cost estimates and for a given renewable source, we identify the optimal investments by selecting the case with the lowest cost. Then, we separately provide a 50% subsidy for each conventional source and compare the new investment levels against the original investments. We report our main findings in Figure 6 (see Appendix C for details).

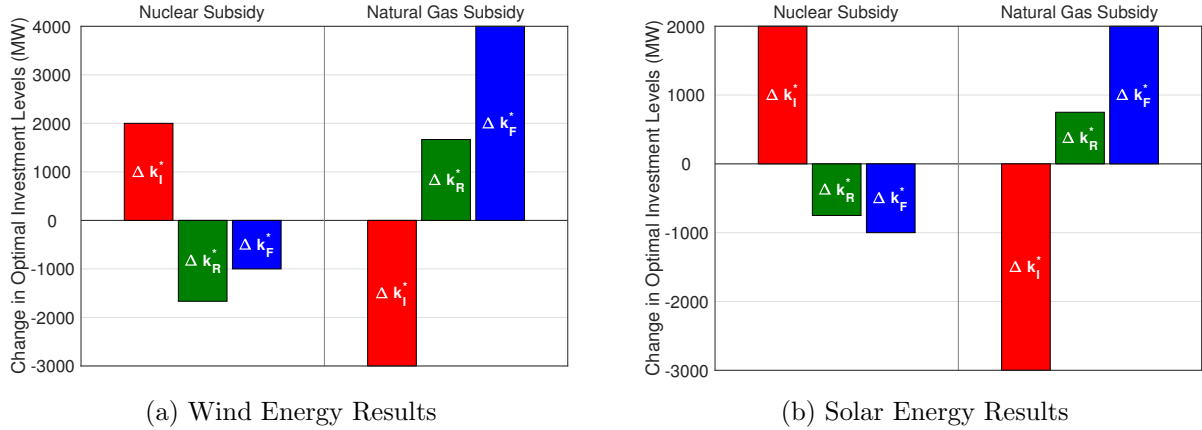(a) Wind Energy Results   (b) Solar Energy Results

Figure 6: Effect of Subsidies

Figure 6 plots the change in the optimal investment levels, compared with the original investments, when a subsidy is provided for a conventional source. For the renewable sources, we report the effective investment level that accounts for the intermittency of wind and solar energy by multiplying its optimal investment level with its capacity factor as described above. In Figure 6a, we consider wind energy as the renewable source. In the left panel of this figure, we observe that providing a subsidy for the nuclear energy source leads to an increase in the capacity of that source and

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a decrease in the capacity of wind and flexible sources. In the right panel in Figure 6a, we observe that a subsidy for the natural gas results in a lower investment in the inflexible source, but investments in other sources increase. Figure 6b presents similar results when solar energy is considered as the renewable source. These results validate Proposition 3 because the same complementarity and substitution effects are found between energy sources.

To sum, in this case study, we use a practical dispatch model to refine our definition of operational flexibility. We observe that our insights continue to hold. That is, renewable and flexible sources are complements, whereas renewable and inflexible sources are substitutes in a realistic setting that is not subject to the limitations of our analytical model.

7 Discussion of Modeling Assumptions

7.1 Spot Market

In our main model given in (3)–(6), we consider a vertically integrated utility firm that does not participate in a spot market to buy or sell electricity. In practice, more than half of U.S. utility firms use spot markets, such as the real-time Energy Imbalance Market in the SPP (EIA 2011a). In this section, we consider the effect of a spot market on the capacity investments of a utility firm.

In an electricity market, a utility firm can procure electricity either from its own generation sources (self-schedule) or from other suppliers through bilateral contracts and spot markets (FERC 2015b, p. 62). The most common way for a utility firm to procure electricity is self-schedule. For example, in the largest electricity market of the U.S. (PJM Interconnect), utility firms have generated more than 60% of their electricity from their own sources in 2014 (Monitoring Analytics 2015, p. 97). The remaining electricity can be purchased from a spot market in which the price varies stochastically. Furthermore, this market, such as the one in PJM, has a relatively low volume so that the price might be affected by the amount of electricity traded. Considering these factors, we assume that the utility firm faces the following price in the spot market:

$$p_S^n\left(\Gamma, q_S\right) = \Gamma + \frac{b_n}{2}q_S, \tag{12}$$

where Γ is a random variable representing price uncertainty, q_S is the amount of electricity bought by the utility firm, and $b_n > 0$ is the price responsiveness parameter in period n. We note that q_S is

negative if the utility firm sells electricity in the market, which causes the market price to decrease. On the other hand, if the utility firm buys electricity from the market, q_S is positive, which causes the market price to increase. We note that our results hold for any positive b_n, that is, our results are robust to the magnitude of the effect of the utility firm on the market price. Similar models for spot markets are considered in the literature (c.f., Martínez-de-Albéniz and Simchi-Levi 2005).

In the presence of the spot market, we modify the second stage of the utility firm's problem as

$$C_n\left(\boldsymbol{k}, \xi, \theta, \gamma\right) = \min_{q_R, q_F \geq 0, q_S \in \mathbb{R}} c_F q_F + p_S^n\left(\gamma, q_S\right) q_S \tag{13}$$

$$\text{subject to} \quad q_R \leq \theta k_R \tag{14}$$

$$q_F \leq k_F \tag{15}$$

$$q_S = \xi - k_I - q_R - q_F. \tag{16}$$

Following Lemma 1 that it is optimal for the utility firm to dispatch the entire inflexible capacity at every period, the utility firm minimizes its generation and market transaction cost based on the dispatch levels of the renewable and flexible sources, as well as the quantity traded in the spot market (q_S). In this stage, the utility firm observes the forecast of Γ as γ. Furthermore, q_S is defined in (16) as the difference between the demand level and the dispatched electricity from the utility firm's own investments. Recall that q_S is negative if the firm sells electricity in the market. In this case, the second term in (13), i.e., $p_S^n\left(\gamma, q_S\right) q_S$, is also negative, indicating a decrease in the cost for the utility firm. On the other hand, if q_S is positive, the utility firm buys electricity from the market, and the second term in (13) is positive, indicating an increase in the cost for the utility firm. With this per period cost, the first stage problem is

$$\min_{\boldsymbol{k} \in \mathbb{R}_+^3} \bar{\bar{\Pi}}\left(\boldsymbol{k}\right) = \left(\alpha_I + c_I N\right) k_I + \alpha_R k_R + \alpha_F k_F + E\left[\sum_{n=1}^N C_n\left(\boldsymbol{k}, \Xi_n, \Theta_n, \Gamma_n\right)\right]. \tag{17}$$

We next derive the optimal dispatch policy. In this case, in addition to supply and demand uncertainties, we also consider a spot price uncertainty, which complicates our analysis considerably.

Lemma 3. *Consider the dispatch problem given in (13)–(16). (i) The optimal dispatch policy is to set* $q_R^*\left(\boldsymbol{k}, \xi, \theta, \gamma\right) = \min\left(\theta k_R, \xi - k_I + \frac{\gamma}{b_n}\right)^+$ *and* $q_F^*\left(\boldsymbol{k}, \xi, \theta, \gamma\right) = \min\left(k_F, \xi - k_I - \theta k_R + \frac{\gamma - c_F}{b_n}\right)^+$. *(ii) The first stage problem given in (17) is convex in* $\boldsymbol{k}$.

Lemma 3 is the extension of Lemma 2 to the spot market setting. In this case, in addition

to the demand and intermittency forecasts, the optimal dispatch levels also depend on the market price forecast. If the forecast of the market price is too low (γ is small), neither the renewable nor the flexible source is dispatched. That is, unlike the main model, the utility firm might find it optimal not to use all the renewable energy capacity in all periods. As the market price becomes higher, the renewable source, followed by the flexible source, is dispatched. We also note that the dispatch level of the inflexible source is equal to its capacity as in the main model.[6]

Next, we present our main result that identifies the interactions between energy sources in the spot market setting. We continue to consider the interior solution case (i.e., $\boldsymbol{k}^* > 0$). To identify the interactions, we use the following assumption as a sufficient condition.

Assumption 2. *(i) The utility firm dispatches all of its available renewable energy in each period, i.e., $q_R^* = \Theta_n k_R$. (ii) Demand, intermittency, and market price uncertainties are independent of one another. (iii) Demand distribution Ξ_n is bounded above by a constant κ_n. (iv) Intermittency uncertainty Θ_n follows a stationary Bernoulli distribution, where $\Theta_n = 1$ with probability q, and $\Theta_n = 0$ with probability $1 - q$. (v) Market price uncertainty Γ_n follows a uniform distribution between L_n and U_n such that $L_n \leq -b_n \kappa_n$.*

Assumption 2(i) ignores the possibility that the utility firm does not use (i.e., curtails) its renewable source. This is a good approximation of the practice because curtailment as a fraction of wind capacity is less than 4% in the U.S. in 2014 (Bird et al. 2014). The second part of the assumption is mainly required to establish the complementarity result between the renewable and flexible sources. In the absence of this assumption, we numerically observe that our results still hold. Assumption 2(iii) bounds the demand distribution from above. This is not very restrictive because such a distribution can be closely approximated by an unbounded random variable (e.g. Normal) as long as κ_n is large enough compared to the variance (Petruzzi and Dada 1999). Assumption 2(iv) imposes a Bernoulli intermittency distribution. This is a sufficient (but not necessary) condition, and it is commonly used in the literature for the intermittency of renewables (c.f., Aflaki and Netessine 2017, Baranes et al. 2017, and Kök et al. 2018). The last part of the assumption suggests that the market price follows a nonstationary uniform distribution, and it can be negative. Note

---

[6]Based on this optimal dispatch policy, optimal investment levels can be characterized similar to the multi-dimensional newsvendor solution in the main model. In addition, the cross effects of subsidies are equivalent, i.e., $\frac{dk_i^*}{d\alpha_j} = \frac{dk_j^*}{d\alpha_i}, \forall i, j \in \{I, R, F\}$. The proofs of these results are available from the authors.

that negative prices are observed in practice (c.f. Zhou et al. 2015).

Proposition 4. *(i) The inflexible and renewable sources are substitutes. If Assumption 2 holds, then (ii) the inflexible and flexible sources are substitutes, and (iii) the renewable and flexible sources are complements.*

Proposition 4 shows that our main insight also holds when a spot market is considered under certain sufficient conditions. That is, the relationship between a renewable and a conventional source is determined by operational flexibility. If the conventional source is inflexible, it substitutes the renewable source; otherwise, it complements the renewable source.

7.2 Oversupply Penalty

In our model, the utility firm incurs an explicit penalty cost in the case of undersupply, i.e., if the electricity demand exceeds the electricity supply. In this subsection, we extend our model by considering an explicit oversupply penalty cost due to technical issues, such as transmission congestion (Bird et al. 2014, p. 1). Another reason for the oversupply penalty is the cost of reducing the output of conventional sources (leading to cycling costs, see Bird et al. 2014, p. 13). We observe that our main conclusion on the substitution and complementarity effects continues to hold under an explicit oversupply penalty.

To model oversupply penalty, we modify the second stage dispatch problem as

$$C\left(\boldsymbol{k}, \xi, \theta\right) = \min_{0 \leq q_R \leq \theta k_R, 0 \leq q_F \leq k_F} c_F q_F + r_u \left(\xi - k_I - q_R - q_F\right)^+ + r_o \left(k_I + q_R + q_F - \xi\right)^+, \quad (18)$$

where r_u and r_o denote the undersupply and the oversupply penalty rate, respectively. In this case, the optimal dispatch policy is the same as that of the main model: all the inflexible capacity is dispatched at each period, and the renewable source is used before the flexible source. Under the optimal dispatch policy, given in Appendix D, the oversupply penalty only occurs if the demand level is less than the capacity of the inflexible source. This is because the utility firm dispatches the renewable and flexible sources based on the five-minute-ahead demand and intermittency forecasts, which are assumed to be accurate as in Wu and Kapuscinski (2013). Hence, in the optimal dispatch policy, the renewable and flexible sources never cause an oversupply penalty. Based on the optimal dispatch policy, we characterize the optimal capacity portfolio in Appendix D as a multidimensional

newsvendor solution.

Proposition 5. *An increase in the oversupply penalty rate r_o leads to a lower investment level in the inflexible source. If Assumption 1 holds, then an increase in r_o leads to a higher investment level in the renewable and flexible sources.*

Proposition 5 suggests that an increase in the oversupply penalty leads to a lower investment in the inflexible source but a higher investment in the other sources. This is because only the inflexible source incurs the oversupply penalty in the optimal capacity portfolio, as explained above. We next investigate the relationship between energy sources under the oversupply penalty.

Proposition 6. *(i) The inflexible and renewable sources are substitutes. If Assumption 1 holds in the strict sense, then (ii) the inflexible and flexible sources are substitutes, and (iii) there exists $\bar{r} > 0$ such that if $r_o \leq \bar{r}$, then the renewable and flexible sources are complements.*

Proposition 6(i) shows that the inflexible and renewable sources remain to be substitutes under an oversupply penalty. The inflexible and flexible sources are also substitutes if Assumption 1 holds in the strict sense, i.e., $g(\xi, \theta)$, defined in (10), is strictly log-concave in ξ for any θ, and $\frac{g(\xi, \theta_2)}{g(\xi, \theta_1)}$ is strictly decreasing in ξ for any $\theta_2 \geq \theta_1$. Moreover, this assumption is satisfied by wind energy (see Figure 4), and the complementarity effect between the flexible source and wind energy holds if the oversupply penalty rate is sufficiently low, i.e., $r_o \leq \bar{r}$. We cannot analytically characterize $\bar{r}$; however, in numerical studies, we observe that $\bar{r}$ is approximately \$200 per kWh. In practice, r_o is capped at \$5 per kWh in Texas (Cohen 2012, p. 184), indicating that r_o is well below $\bar{r}$ so that the complementarity result holds. To illustrate our findings, we present a numerical study in Figure 7a based on the Texas data. We observe that the same complementarity and substitution effects hold as in the case without an oversupply penalty (see Figure 5a) for wind energy.

In the case of solar energy, Assumption 1 is not satisfied; hence, we present a numerical study in Figure 7b. We observe that the range of α_F values for which solar energy and the flexible source are substitutes is expanded compared to Figure 5b (where $r_o = 0$). Nevertheless, our main conclusion for solar energy continues to hold under the oversupply penalty: solar energy and the flexible source are complements as long as the subsidy level is high for the flexible source (i.e., α_F is low).
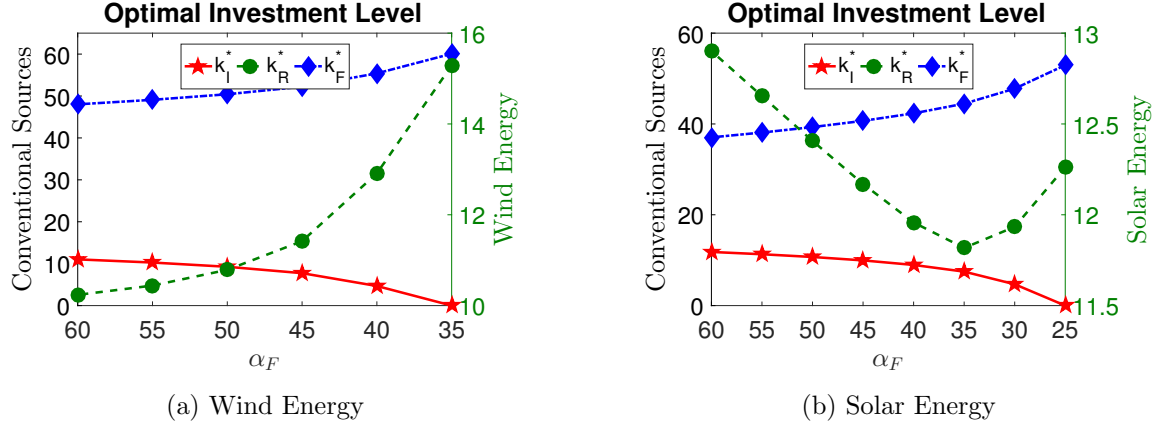
(a) Wind Energy   (b) Solar Energy

Figure 7: Effect of a Subsidy for the Flexible Source

7.3 Effects of Carbon Tax Policy

To increase investment in renewable sources, 45 countries have adopted policies that penalize carbon emissions (World Bank 2018, p. 17). One such policy is carbon tax. In this subsection, we investigate how the carbon tax policy affects investment in energy sources. Specifically, we present an extension of our model by considering a tax level given as t. Let the emission intensity of a source be e_i for $i \in \{I, F\}$. Here, $e_R = 0$ because a renewable source (e.g., wind) does not produce emissions. Under the carbon tax, the generation cost of the flexible and inflexible sources become $c_F + te_F$ and $c_I + te_I$ in (3) and (6), respectively.

We can characterize the optimal capacity portfolio (similar to Proposition 1) under the carbon tax. However, we are not able to analytically establish the effect of the tax on the optimal investment levels, i.e., dk_i^*/dt for each $i \in \{I, R, F\}$, for a general demand and intermittency distribution. Thus, to study carbon tax, we resort to a numerical analysis calibrated by the Texas data.

Figure 8 illustrates the effect of carbon tax t on the energy investments for two different inflexible sources. First, in Figure 8a, we consider that the inflexible source is carbon-free nuclear energy ($e_I = 0$), which is not affected by the carbon tax. The tax increases the cost of generating electricity from the flexible source because $e_F > 0$. As a result, the optimal investment in the flexible source decreases, which leads to a lower renewable energy investment due to the complementarity effect. Second, in Figure 8b, we consider coal power as the inflexible source. In this case, $e_I > e_F$, and the carbon tax leads to a higher investment in the flexible and renewable sources. Therefore, the previous claims that carbon tax leads to a higher renewable energy investment only holds if the

(a) Nuclear Energy ($e_F > e_I = 0$)
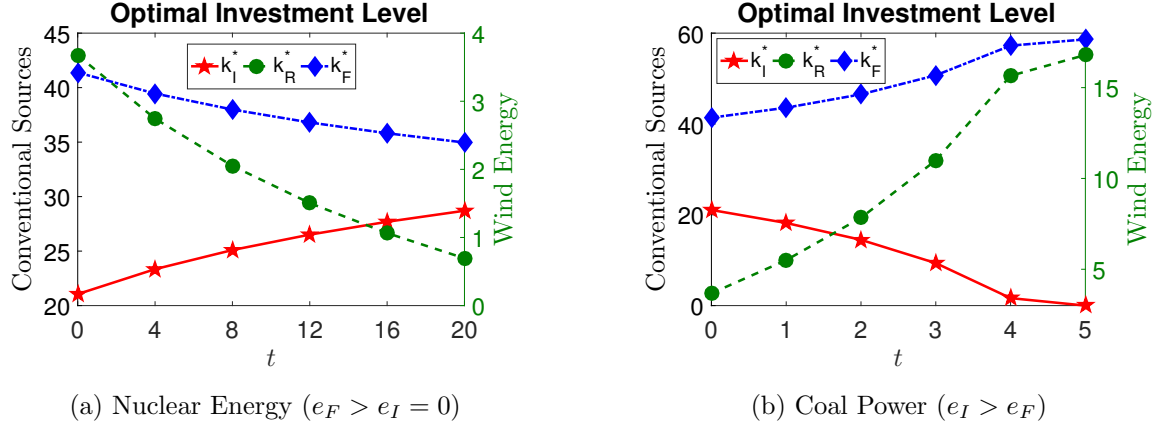
(b) Coal Power ($e_I > e_F$)

Figure 8: Effect of Carbon Tax on the Optimal Investment Levels for Different Inflexible Sources

inflexible source is more carbon-intensive than the flexible source. On the other hand, carbon tax leads to a lower investment in renewable energy if the inflexible source is carbon-free (e.g., nuclear), but the flexible source is carbon-intensive (e.g., natural gas).

We next validate the insights on the effects of carbon tax policy by using the case study presented in Section 6. Specifically, we impose a carbon tax of \$10 per ton of CO_2 in the unit commitment and dispatch model (UCDM) and, by following similar steps as in Section 6, we find the optimal investment levels. We then compare the investment levels under the tax to the original investments without the tax. We note that our findings are robust to the carbon tax level.
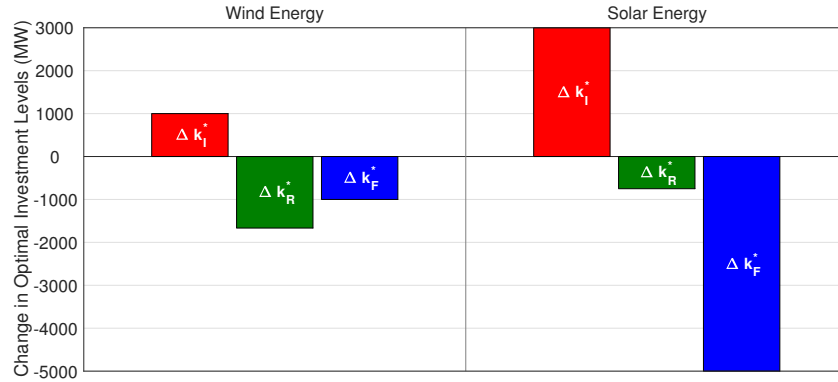

Figure 9: Effect of Carbon Tax

Figure 9 plots the change in the optimal investment levels when a carbon tax is imposed. On the left panel, we consider wind energy as the renewable source; on the right panel, solar energy is the renewable source. In both cases, the inflexible source is carbon-free nuclear energy, whereas the flexible source is carbon-emitting natural gas. Figure 9 shows that the carbon tax leads to a

lower renewable energy investment, validating the results of the analytical model (see Figure 8).

7.4   Energy Efficiency and Demand Response

Our model considers capacity investment in energy sources, which is related to managing the supply side of energy systems. Some of the incentives that reduce demand, such as Energy Efficiency (EE) and Demand Response (DR), can be incorporated into the current model. Specifically, EE refers to the incentives that a utility firm provides to its customers so that these customers reduce their total electricity demand by using more efficient devices. For example, Duke Energy, through its Appliance Recycling Program, offers a rebate to those who want to replace their old refrigerators with more efficient ones (Duke Energy 2015a). DR, on the other hand, aims to reduce the demand only during peak demand periods. For instance, MidWest Energy compensates farmers who curtail the usage of water pumps upon a service call during high-demand hours (Midwest Energy 2015).

From the perspective of a utility firm, EE and DR are equivalent to the inflexible and flexible sources, respectively. Specifically, the rebate paid to customers under EE corresponds to the investment cost of the inflexible source, and the curtailment payments made under DR correspond to the generation cost of the flexible source. Furthermore, similar to nuclear power, EE is used to reduce the baseload demand, whereas, similar to natural gas, DR is used to reduce demand at high-demand periods. In terms of cost, the rebates given under EE are one-time payments similar to the investment cost of the inflexible source. In contrast, DR involves a low initial cost but high recurring curtailment payments similar to the high generation cost of the flexible source. Finally, similar to the flexible source, DR contracts are capable of curtailing the demand within seconds because they involve automated response (DOE 2011). Thus, our model indicates that EE and renewable energy investment are substitutes, whereas DR and renewable investment are complements. As the share of renewables increases, the need for and the importance of DR will increase, as well.

7.5   Dual Sourcing

Throughout the paper, we assume that the triple sourcing strategy is optimal, i.e., $\boldsymbol{k}^* > 0$. In some cost parameters, a dual sourcing strategy may be optimal (e.g., $k_I^*, k_R^* > 0$ and $k_F^* = 0$). In any dual sourcing case, the two sources included in the optimal portfolio are substitutes. The details

of the proof are available from the authors. We note that this conclusion is the same as that of the dual sourcing literature (c.f., Sting and Huchzermeier 2012).

8   Conclusion

In this paper, we consider the capacity investments of a utility firm in renewable and conventional sources with different levels of operational flexibility. We characterize the optimal investment levels and determine the role of operational flexibility in identifying the interaction between energy sources. Specifically, a renewable and a conventional source are substitutes (complements, respectively) if the conventional source is inflexible (flexible, respectively). We validate this result by using real electricity generation and demand data from Texas.

This paper has significant policy implications and can provide guidelines for designing policies to promote renewables. First, we show that from the perspective of a utility firm, the intermittency problem can be alleviated by flexible energy sources, such as open-cycle natural gas-fired power plants. Thus, low natural gas prices may promote investment in renewables. Second, policymakers should refrain from providing a subsidy for an inflexible source (e.g., nuclear or coal power) because this subsidy leads to a lower investment in renewables. Finally, a carbon tax is only effective in increasing renewable investment if the inflexible source is carbon-intensive, such as coal power. Thus, given the high share of nuclear energy as the inflexible source in the U.S., the tax might not lead to increased renewable investment.

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Online Supplement to "Investments in Renewable and Conventional Energy: The Role of Operational Flexibility"

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A Evaluation of $g(\xi, \theta)$

In this section, we introduce a serially correlated process for the demand and intermittency uncertainty. This process consists of three components: trend, seasonality, and a vector autoregressive noise. Based on this process, we show that the joint distribution of demand and intermittency (Ξ_n, Θ_n) in each period n is a bivariate normal distribution. By estimating the mean and covariance matrix of these distributions, we evaluate the $g(\xi, \theta)$ function given in (10).

Let the joint demand and intermittency process be

$$
\begin{bmatrix} \Xi_n \\ \Theta_n \end{bmatrix} = \begin{bmatrix} d_{\Xi,n} \\ d_{\Theta,n} \end{bmatrix} + \begin{bmatrix} s_{\Xi,n} \\ s_{\Theta,n} \end{bmatrix} + \begin{bmatrix} Y_n \\ Z_n \end{bmatrix},
\tag{A.1}
$$

where $d_{\Xi,n}$ and $s_{\Xi,n}$ denote the trend and the seasonality in demand distribution in period n, respectively. Similarly, $d_{\Theta,n}$ and $s_{\Theta,n}$ are the trend and seasonality in intermittency. Furthermore, $\{Y_n, Z_n\}$ follows a vector autoregressive (VAR) model of order 1:

$$
\begin{bmatrix} Y_n \\ Z_n \end{bmatrix} = \Phi \begin{bmatrix} Y_{n-1} \\ Z_{n-1} \end{bmatrix} + \begin{bmatrix} w_n \\ z_n \end{bmatrix},
\tag{A.2}
$$

where $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$ and $\{w_n, z_n\}$ follows a bivariate normal distribution, i.e., $\{w_n, z_n\} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma \right)$ with $\Sigma = \begin{bmatrix} \sigma_w^2 & \rho\sigma_w\sigma_z \\ \rho\sigma_w\sigma_z & \sigma_z^2 \end{bmatrix}$. In this case, according to Huang and Schneider (2011), the stationary distribution of $\{Y_n, Z_n\}$ can be shown to be bivariate normal

$$
\begin{bmatrix} Y_n \\ Z_n \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \right),
\tag{A.3}
$$

where $\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$ is the solution of the Lyapunov equation: $\Lambda = \Phi\Lambda\Phi' + \Sigma$. Hence, the

joint demand and intermittency distribution is given as

$$\begin{bmatrix} \Xi_n \\ \Theta_n \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} d_{\Xi,n} + s_{\Xi,n} \\ d_{\Theta,n} + s_{\Theta,n} \end{bmatrix}, \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \right). \tag{A.4}$$

We estimate each parameter in the above bivariate normal distribution for each period n over a year. That is, we set $N = 104,832$ (five-minute intervals in 364 days). By using the realized demand and intermittency data from the period of 2016–2018 in the Southwest Power Pool (SPP), we first estimate the trend components $d_{\Xi,n}$ and $d_{\Theta,n}$ and the seasonality components $s_{\Xi,n}$ and $s_{\Theta,n}$ for all n. After de-trending and de-seasonalizing, the remaining data follows the VAR(1) process given in (A.3). Hence, we solve the Lyapunov equation to find Λ. This completes the estimation process for the parameters in (A.4). By using these parameters for each period n, we characterize $f_{(\Xi_n, \Theta_n)}(\cdot, \cdot)$; in return, $g(\xi, \theta)$ can be computed for any given ξ and θ.

We finally note that the above estimation procedure is based on an additive trend and seasonality component. As a result of this additive formulation, the covariance matrix for each (Ξ_n, Θ_n) is constant and given as Λ in (A.4). We have also considered a multiplicative seasonality form and observed that the covariance matrix becomes time-dependent in this case. Moreover, under this multiplicative form, Assumption 1 is still reasonable for wind energy.

B   Parameter Values in Numerical Examples

In Figure 5, we consider a peak and an off-peak period, i.e., $N = 2$. We estimate the parameters of the bivariate demand and supply distributions by using the data from Texas. The average demand during the peak and off-peak period is 40.84 GW and 32.00 GW, respectively, with a standard deviation of 10.70 GW and 6.62 GW, respectively. The correlation coefficients between the demand and supply distributions are 0.57 for wind energy and 0.25 for solar energy during the peak period and 0.35 for wind energy and 0.18 for solar energy during the off-peak period. Finally, we use similar cost parameters as in the case study and report the effective investment levels in renewable sources by accounting for intermittency.

Figure 5a illustrates that without a subsidy for the flexible source ($\alpha_F = 50$), the share of the inflexible, renewable, and flexible sources in the capacity portfolio is 38%, 6%, and 56%, respectively, in Texas. These values are similar to the actual shares given respectively as 29%, 10%, and 62%

(based on the case study data).

In Figure 7, we use the same parameter values as in Figure 5 and $r_u = r_o = \$5$ per kWh.

Finally, in Figure 8, wind energy is the renewable source, and we continue to let $N = 2$. We use $e_F = 1.21$ and $e_I = 2.07$ (for coal) pounds of CO_2 per kWh (EIA 2016). Note that compared to panel (a), we report a limited range of carbon tax t levels in panel (b). This is because for high t values, coal power becomes uneconomic, and the utility firm does not invest in it.

C   Details of the Case Study

To estimate the investment cost parameters, we use the Transparent Cost Database (TCDB, available at http://en.openei.org/apps/TCDB/), which tracks various publications that estimate cost figures for power plants. The estimate for the overnight capital cost of wind and solar energy varies significantly throughout the years. For wind energy, the median estimate is $\$1.57$M per MW, whereas the most recent one is $\$1.73$M per MW. Based on this data, for wind energy, we set $\alpha_R = \$1.65$M per MW. For solar energy, we let $\alpha_R = \$1$M per MW, which is at the lower end of the estimates. For nuclear source, there is limited data in the TCDB. Thus, we rely on a report of the Energy Information Administration (EIA 2013) and observe that the overnight capital cost is $\$5.53$M per MW. Furthermore, we adjust these cost figures based on differences in the economic life of power plants. For example, we assume that the economic life of a nuclear power plant is twice that of a wind plant and set $\alpha_I = \$5.53/2 \approx \$3$M per MW. Finally, for the open-cycle gas turbine, we set $\alpha_F = \$50,000$ per MW. This is because, in the TCDB, the cost estimates are as low as $\$200,000$ per MW, and gas turbines also have a much longer economic life than renewables.

Table 3: Optimal Investment Levels (MW) in the Case Study

Optimal Investment Level	Original Costs	Wind Energy		Solar Energy	
		Nuclear Subsidy	Natural Gas Subsidy	Nuclear Subsidy	Natural Gas Subsidy
k_I^*	3,000	5,000	0	5,000	0
k_R^*	5,000	0	10,000	0	10,000
k_F^*	1,000	0	5,000	0	3,000

Table 3 reports the optimal investment levels (in addition to the existing fleet) under original costs and conventional energy subsidies when wind or solar energy is considered as the renewable source.

Finally, in analyzing the effect of a carbon tax, we set $t = \$10$ per ton of CO_2. This figure is consistent with the average carbon tax level in 2010 prices (see https://www.carbontax.org/where-carbon-is-taxed/).

D   Oversupply Penalty

To characterize the optimal capacity investments under the oversupply penalty case, first recall the second stage problem given in (18) as

$$C\left(\boldsymbol{k}, \xi, \theta\right) = \min_{0 \leq q_R \leq \theta k_R, 0 \leq q_F \leq k_F} c_F q_F + r_u \left(\xi - k_I - q_R - q_F\right)^+ + r_o \left(k_I + q_R + q_F - \xi\right)^+.$$

In this formulation, the optimal dispatch policy identified in Lemma 1 and 2 continues to hold, i.e., $q_I^* = k_I$, $q_R^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(\theta k_R, \xi - k_I\right)^+$, and $q_F^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(k_F, \xi - k_I - \theta k_R\right)^+$. As in Section 4, we can construct the dual of this dispatch problem such that $\lambda_i^*\left(\boldsymbol{k}, \xi, \theta\right)$ denotes the dual variable of the capacity constraint for source i. Furthermore, we can show that $\nabla_{\boldsymbol{k}} E[C(\boldsymbol{k}, \Xi_n, \Theta_n)] = -E[\boldsymbol{\lambda}(\boldsymbol{k}, \Xi_n, \Theta_n)]$ for all n. These expectations should be computed slightly differently under the oversupply penalty case. Specifically, in Figure 10, we illustrate the intermittency and demand space partitions over which these expectations need to be computed. We next compare Figure 10
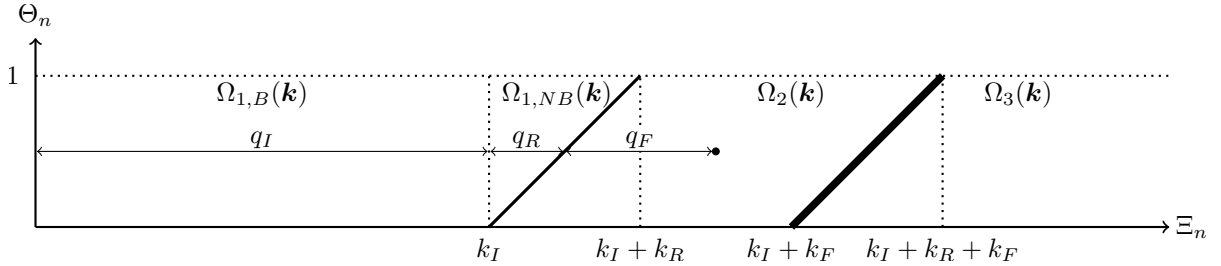

Figure 10: Partitions of Demand and Intermittency Space under Oversupply Penalty
Notes. In Figure 10, for illustration purposes, we assume that $k_F > k_R$.

with Figure 3, i.e., the corresponding figure in the main model. In Figure 10, Ω_1 is divided into two subregions as $\Omega_{1,B}\left(\boldsymbol{k}\right) = \{(\xi, \theta) | \xi \leq k_I\}$ and $\Omega_{1,NB}\left(\boldsymbol{k}\right) = \{(\xi, \theta) | k_I < \xi \leq k_I + \theta k_R\}$. In $\Omega_{1,NB}\left(\boldsymbol{k}\right)$ capacity constraints are nonbinding for all sources, whereas in $\Omega_{1,B}$, there is an excess inflexible capacity, causing $\lambda_I^*\left(\boldsymbol{k}, \xi, \theta\right) = -r_o$. By accounting for this difference, we characterize the optimal capacity portfolio below.

Proposition 7. *Consider the problem given in* (18) *and* (6). *An investment vector $\boldsymbol{k}^* \in \mathbb{R}_+^3$ is*

optimal if and only if there exists a $\boldsymbol{v} \in \mathbb{R}_+^3$ such that

$$\sum_{n=1}^{N} \left(E \left[\begin{array}{c} -r_o \\ 0 \\ 0 \end{array} \middle| \Omega_{1,B}\left(\boldsymbol{k}^*\right) \right] P^n\left(\Omega_{1,B}\left(\boldsymbol{k}^*\right)\right) + E \left[\begin{array}{c} c_F \\ \Theta_n c_F \\ 0 \end{array} \middle| \Omega_2\left(\boldsymbol{k}^*\right) \right] P^n\left(\Omega_2\left(\boldsymbol{k}^*\right)\right) \right. \tag{D.1}$$

$$\left. + E \left[\begin{array}{c} r_u \\ \Theta_n r_u \\ r_u - c_F \end{array} \middle| \Omega_3\left(\boldsymbol{k}^*\right) \right] P^n\left(\Omega_3\left(\boldsymbol{k}^*\right)\right) \right) = \left(\begin{array}{c} \alpha_I + c_I N - v_I \\ \alpha_R - v_R \\ \alpha_F - v_F \end{array} \right) \tag{D.2}$$

$$\forall i \in \{I, R, F\} : k_i v_i = 0 \tag{D.3}$$

Proposition 7 characterizes the KKT conditions for the optimal capacity investments under the oversupply penalty rate. Compared to the KKT conditions in Proposition 1, the conditions in this case include the oversupply penalty for the inflexible source when the demand exceeds the capacity of the inflexible source in $\Omega_{1,B}\left(\boldsymbol{k}^*\right)$. The reason is that the oversupply penalty is only incurred by the inflexible source as the other sources would not be dispatched to avoid any oversupply penalty if the demand exceeds their capacity.

E Proofs

Proof of Lemma 1. Let $q_I^* < k_I^*$ be the optimal solution in (2). Define $\bar{k}_I = k_I^* - \epsilon$ for some $\epsilon > 0$ such that $q_I^* < \bar{k}_I$. Note that $\bar{k}_I$ is a feasible solution with a strictly lower cost, contradicting the optimality of k_I^*. □

Proof of Lemma 2. Observe that (3) decreases in q_R and q_F at respective rates of r and $r - c_F$. Hence, it is optimal to dispatch the renewable source first, followed by the flexible source, i.e., $q_R^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(\theta k_R, \xi - k_I\right)^+$ and $q_F^*\left(\boldsymbol{k}, \xi, \theta\right) = \min\left(k_F, \xi - k_I - \theta k_R\right)^+$. □

Before proceeding to the proofs of the propositions, we present a lemma for later use.

Lemma 4. *Consider a log-concave function $f\left(\cdot\right)$. Let $x, y,$ and z be positive scalars. Then,*

$$\frac{f(x+y)}{f(x)} \geq \frac{f(x+y+z)}{f(x+z)}. \tag{E.1}$$

Proof of Lemma 4. This inequality holds if and only if $\log f\left(x+y\right) + \log f\left(x+z\right) \geq \log f\left(x+y+z\right) +$

$\log f(x)$. Let $\lambda = \frac{y}{y+z}$, by the definition of log-concavity

$$\log f(\lambda (x + y + z) + (1 - \lambda) x = x + y) \geq \lambda \log f(x + y + z) + (1 - \lambda) \log f(x)$$

$$\log f((1 - \lambda) (x + y + z) + \lambda x = x + z) \geq (1 - \lambda) \log f(x + y + z) + \lambda \log f(x).$$

Adding these inequalities side by side, we observe that $\log f(x + y) + \log f(x + z) \geq \log f(x + y + z) + \log f(x)$. Hence, the inequality given in (E.1) holds. $\qquad\square$

Proof of Proposition 1. This proof follows similar arguments as in Van Mieghem (1998) and Sting and Huchzermeier (2012). We first note that the dispatch problem given in (3)–(5) can be equivalently expressed as

$$C(\boldsymbol{k}, \xi, \theta) = \min_{q_R, q_F, s \geq 0} \quad c_F q_F + rs$$

$$\text{subject to} \quad \frac{q_R}{\theta} \leq k_R \leftarrow \lambda_R$$

$$q_F \leq k_F \leftarrow \lambda_F$$

$$-s - q_R - q_F \leq k_I - \xi \leftarrow \lambda_I,$$

where λ_i's are the decision variables of the following dual problem

$$\max_{\boldsymbol{\lambda} \in \mathbb{R}_+^3} (\xi - k_I) \lambda_I - k_R \lambda_R - k_F \lambda_F$$

$$\text{subject to} \quad \lambda_I \leq \frac{\lambda_R}{\theta}$$

$$\lambda_I \leq \lambda_F + c_F$$

$$\lambda_I \leq r.$$

Since $C(\cdot, \xi, \theta)$ is the minimal solution of a linear program, it is convex. Thus, $\bar{\Pi}(\cdot)$ given in (6) is also convex because convexity is preserved under expectation and summation. Therefore, KKT conditions (7) and (8) are sufficient and necessary to identify the minimizer of $\bar{\Pi}(\cdot)$.

We next show that $E[C(\boldsymbol{k}, \Xi, \Theta)] = -E[\boldsymbol{\lambda}(\boldsymbol{k}, \Xi, \Theta)]$, where we drop the period index for brevity. First, we note that the primal problem is always finite as the demand and intermittency distributions are bounded. Hence, the dual problem and the primal has the same objective value when they are both optimal. Let $\boldsymbol{\lambda}^*(\boldsymbol{k}, \xi, \theta)$ be the optimal dual solution for given $\boldsymbol{k}, \xi$, and θ and fix some $\boldsymbol{k}^0 \in \mathbb{R}_+^3$. Then, for any $\boldsymbol{k} \in \mathbb{R}_+^3$: $C(\boldsymbol{k}, \xi, \theta) \geq \xi \lambda_I^*(\boldsymbol{k}^0, \xi, \theta) - \boldsymbol{k}' \boldsymbol{\lambda}^*(\boldsymbol{k}^0, \xi, \theta)$. Combining this with $C(\boldsymbol{k}^0, \xi, \theta) = \xi \lambda_I^*(\boldsymbol{k}^0, \xi, \theta) - \boldsymbol{k}^{0'} \boldsymbol{\lambda}^*(\boldsymbol{k}^0, \xi, \theta)$, we obtain that

$$-C(\boldsymbol{k}, \xi, \theta) \leq -C(\boldsymbol{k}^0, \xi, \theta) + (\boldsymbol{k} - \boldsymbol{k}^0)' \boldsymbol{\lambda}^*(\boldsymbol{k}^0, \xi, \theta).$$

Taking expectations of both sides, we observe that $E[\boldsymbol{\lambda}\left(\boldsymbol{k}, \Xi, \Theta\right)]$ is a subgradient of $E[-C\left(\boldsymbol{k}, \Xi, \Theta\right)]$ evaluated at $\boldsymbol{k}^0$. Because $C\left(\boldsymbol{k}, \Xi, \Theta\right)$ is convex, it is differentiable almost everywhere (a.e.) except for a set whose Lebesgue mesaure is zero as demand and intermittency are continuous. Thus, $\nabla_{\boldsymbol{k}} C\left(\boldsymbol{k}, \Xi, \Theta\right)$ is single-valued a.e. This implies that the subgradient is unique for any $\boldsymbol{k} \in \mathbb{R}_+^3$ and the KKT conditions given in (7) and (8) jointly define the optimal solution. $\qquad\square$

Proof of Proposition 2. Note that in the triple sourcing case (i.e., $\boldsymbol{k} > 0$), by expressing the demand and intermittency partitions explicitly, the first order conditions (FOCs) with respect to (wrt) k_I, k_R, and k_F can be sequentially written as

$$\mathcal{F}\left(\boldsymbol{k}\right) = -\sum_{n=1}^{N} \int_{\theta=0}^{1} \left[r - (r - c_F) F_{\Xi_n|\Theta_n}\left(k_I + \theta k_R + k_F|\theta\right) - c_F F_{\Xi_n|\Theta_n}\left(k_I + \theta k_R|\theta\right)\right] f_{\Theta_n}\left(\theta\right) d\theta + \alpha_I + c_I N$$

$$\mathcal{G}\left(\boldsymbol{k}\right) = -\sum_{n=1}^{N} \int_{\theta=0}^{1} \theta \left[r - (r - c_F) F_{\Xi_n|\Theta_n}\left(k_I + \theta k_R + k_F|\theta\right) - c_F F_{\Xi_n|\Theta_n}\left(k_I + \theta k_R|\theta\right)\right] f_{\Theta_n}\left(\theta\right) d\theta + \alpha_R$$

$$\mathcal{H}\left(\boldsymbol{k}\right) = -\sum_{n=1}^{N} \int_{\theta=0}^{1} (r - c_F) \left(1 - F_{\Xi_n|\Theta_n}\left(k_I + \theta k_R + k_F|\theta\right)\right) f_{\Theta_n}\left(\theta\right) d\theta + \alpha_F.$$

Here, $F_{\Xi_n|\Theta_n}\left(\cdot|\cdot\right)$ denotes the (conditional) cumulative distribution function. In addition, let

$$X_n = (r - c_F) f_{\Xi_n|\Theta_n}\left(k_I^* + \Theta_n k_R^* + k_F^*| \Theta_n\right) \tag{E.2}$$

$$Y_n = c_F f_{\Xi_n|\Theta_n}\left(k_I^* + \Theta_n k_R^*| \Theta_n\right), \tag{E.3}$$

where $E\left[X_n\right] = (r - c_F) \int_{\theta=0}^{1} f_{(\Xi_n, \Theta_n)}\left(k_I^* + \theta k_R^* + k_F^*, \theta\right) d\theta$ and $E\left[Y_n\right] = c_F \int_{\theta=0}^{1} f_{(\Xi_n, \Theta_n)}\left(k_I^* + \theta k_R^*, \theta\right) d\theta$.

(i) We first show that $\frac{dk_I^*}{d\alpha_I} \leq 0$. By implicit differentiation and Cramer's rule, it can be shown that

$$\frac{dk_I^*}{d\alpha_I} = \frac{\begin{vmatrix} -\frac{\partial \mathcal{F}}{\partial \alpha_I} & \frac{\partial \mathcal{F}}{\partial k_R} & \frac{\partial \mathcal{F}}{\partial k_F} \\ -\frac{\partial \mathcal{G}}{\partial \alpha_I} & \frac{\partial \mathcal{G}}{\partial k_R} & \frac{\partial \mathcal{G}}{\partial k_F} \\ -\frac{\partial \mathcal{H}}{\partial \alpha_I} & \frac{\partial \mathcal{H}}{\partial k_R} & \frac{\partial \mathcal{H}}{\partial k_F} \end{vmatrix}}{H}, \tag{E.4}$$

where the FOCs and the determinant of the Hessian matrix, H, are evaluated at $\boldsymbol{k}^*$. Because $H > 0$, to show that $\frac{dk_I^*}{d\alpha_I} \leq 0$, it suffices to show that the numerator is negative in (E.4). The numerator is $-E\left[\sum_{n=1}^{N} \Theta_n^2 (X_n + Y_n)\right] E\left[\sum_{n=1}^{N} X_n\right] + E\left[\sum_{n=1}^{N} \Theta_n X_n\right] E\left[\sum_{n=1}^{N} \Theta_n X_n\right]$, where X_n and Y_n are defined in (E.2) and (E.3), respectively. By Cauchy-Schwarz inequality, this is negative. Next, we consider $\frac{dk_R^*}{d\alpha_R}$ and following similar steps as above, one can show that this derivative is given as $-E\left[\sum_{n=1}^{N} Y_n\right] E\left[\sum_{n=1}^{N} X_n\right]/H$, and hence negative. Finally, $\frac{dk_F^*}{d\alpha_F} = \left\{-E\left[\sum_{n=1}^{N} (X_n + Y_n)\right] E\left[\sum_{n=1}^{N} \Theta_n^2 (X_n + Y_n)\right] + E\left[\sum_{n=1}^{N} \Theta_n (X_n + Y_n)\right] E\left[\sum_{n=1}^{N} \Theta_n (X_n + Y_n)\right]\right\}/H$, which is also negative. (ii) Using similar steps, one can show that $\frac{dk_i^*}{d\alpha_j} = \frac{dk_j^*}{d\alpha_i}$ for $i, j \in \{I, R, F\}$. $\qquad\square$

Proof of Proposition 3. Recall that $H \geq 0$ is the determinant of the Hessian matrix for the objective function given in (6), evaluated at $\boldsymbol{k}^*$. (i) The inflexible and renewable sources are substitutes because $\frac{dk_I^*}{d\alpha_R} = \frac{dk_R^*}{d\alpha_I} = E\left[\sum_{n=1}^N X_n\right] E\left[\sum_{n=1}^N \Theta_n Y_n\right] / H \geq 0$, where X_n and Y_n are defined in (E.2) and (E.3), respectively. (ii) It can be shown that $\frac{dk_F^*}{d\alpha_I} = \frac{dk_I^*}{d\alpha_F} \geq Zc_F(r - c_F) / H$, where $Z = \left(E\left[\sum_{n=1}^N X_n\right] E\left[\sum_{n=1}^N \Theta_n^2 Y_n\right] - E\left[\sum_{n=1}^N \Theta_n X_n\right] E\left[\sum_{n=1}^N \Theta_n Y_n\right]\right) / [c_F(r - c_F)]$. Thus, to show that $\frac{dk_F^*}{d\alpha_I} \geq 0$, it suffices to show that $Z \geq 0$. We next investigate Z by using the definitions of X_n and Y_n, and suppressing the star notation:

$$
Z = \sum_{n=1}^N \int_{\theta=0}^1 f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R + k_F, \theta) d\theta \times \sum_{n=1}^N \int_{\theta=0}^1 \theta^2 f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) d\theta
$$

$$
- \sum_{n=1}^N \int_{\theta=0}^1 \theta f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R + k_F, \theta) d\theta \times \sum_{n=1}^N \int_{\theta=0}^1 \theta f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) d\theta
$$

$$
= \sum_{n=1}^N \sum_{m=1}^N \int_{\theta=0}^1 \int_{\zeta=0}^1 \theta (\theta - \zeta) f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R + k_F, \zeta) d\zeta d\theta
$$

$$
= \sum_{n=1}^N \sum_{m=1}^N \int_{\theta=0}^1 \int_{\zeta=0}^\theta \theta (\theta - \zeta) f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R + k_F, \zeta) d\zeta d\theta
$$

$$
+ \sum_{n=1}^N \sum_{m=1}^N \int_{\theta=0}^1 \int_{\zeta=\theta}^1 \theta (\theta - \zeta) f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R + k_F, \zeta) d\zeta d\theta
$$

Note that, by changing the order of integration, the last double summation is equivalent to $-\sum_{n=1}^N \sum_{m=1}^N \int_{\zeta=0}^1 \int_{\theta=0}^\zeta \theta (\zeta - \theta) f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R + k_F, \zeta) d\theta d\zeta$. In this expression, renaming θ as ζ, m as n and vice-versa, we observe that

$$
Z = \sum_{n=1}^N \sum_{m=1}^N \int_{\theta=0}^1 \left[\int_{\zeta=0}^\theta (\theta - \zeta) \left(\theta f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta) f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R + k_F, \zeta)\right.\right.
$$

$$
\left.\left. - \zeta f_{(\Xi_m, \Theta_m)}(k_I + \zeta k_R, \zeta) f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R + k_F, \theta)\right) d\zeta\right] d\theta.
$$

Here, the summation and integration can be interchanged. Thus, $Z \geq 0$ if

$$
\frac{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \zeta k_R + k_F, \zeta)}{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \zeta k_R, \zeta)} \geq \frac{\zeta}{\theta} \frac{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R + k_F, \theta)}{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta)}.
$$

for all $\theta > \zeta$. This inequality holds under Assumption 1 because

$$
\frac{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \zeta k_R + k_F, \zeta)}{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \zeta k_R, \zeta)} = \frac{g(k_I + \zeta k_R + k_F, \zeta)}{g(k_I + \zeta k_R, \zeta)} \geq \frac{g(k_I + \theta k_R + k_F, \zeta)}{g(k_I + \theta k_R, \zeta)}
$$

$$
\geq \frac{g(k_I + \theta k_R + k_F, \theta)}{g(k_I + \theta k_R, \theta)} = \frac{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R + k_F, \theta)}{\sum_{n=1}^N f_{(\Xi_n, \Theta_n)}(k_I + \theta k_R, \theta)}.
$$

Here, the first inequality is due to Assumption 1(i) that $g(\cdot, \theta)$ is log-concave for any θ. Specifically, apply Lemma 4 with $x \equiv k_I + \zeta k_R$, $y \equiv k_F$, and $z \equiv (\theta - \zeta) k_R$. The second inequality above is due

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to Assumption 1(ii) that $\frac{g(\xi|\theta_2)}{g(\xi|\theta_1)}$ is decreasing in ξ for any $\theta_2 \geq \theta_1$, i.e., $\frac{g(k_I+\theta k_R,\theta)}{g(k_I+\theta k_R,\zeta)} \geq \frac{g(k_I+\theta k_R+k_F,\theta)}{g(k_I+\theta k_R+k_F,\zeta)}$

for $\theta > \zeta$. (iii) Finally, $\frac{dk_F^*}{d\alpha_R} = \frac{dk_R^*}{d\alpha_F} = -Tc_F\left(r - c_F\right)/H$, where H is the Hessian defined above and

$$T = \left(E\left[\sum_{n=1}^{N} X_n\right] E\left[\sum_{n=1}^{N} \Theta_n Y_n\right] - E\left[\sum_{n=1}^{N} \Theta_n X_n\right] E\left[\sum_{n=1}^{N} Y_n\right]\right) / \left[c_F\left(r - c_F\right)\right]. \tag{E.5}$$

Thus, it is sufficient to show that $T \geq 0$ to prove that the flexible source and the renewable source

are complements. Using a similar approach as above, one can show that

$$T = \sum_{n=1}^{N} \sum_{m=1}^{N} \int_{\theta=0}^{1} \left[\int_{\zeta=0}^{\theta} (\theta - \zeta)\left(f_{(\Xi_n,\Theta_n)}(k_I + \theta k_R, \theta)\, f_{(\Xi_m,\Theta_m)}(k_I + \zeta k_R + k_F, \zeta)\right.\right.$$

$$\left.\left. - f_{(\Xi_m,\Theta_m)}(k_I + \zeta k_R, \zeta)\, f_{(\Xi_n,\Theta_n)}(k_I + \theta k_R + k_F, \theta)\right) d\zeta\right] d\theta.$$

The integral and the summation can be interchanged. Consequently, $T \geq 0$, if for all $\theta > \zeta$

$$\frac{\sum_{n=1}^{N} f_{(\Xi_n,\Theta_n)}(k_I + \zeta k_R + k_F, \zeta)}{\sum_{n=1}^{N} f_{(\Xi_n,\Theta_n)}(k_I + \zeta k_R, \zeta)} \geq \frac{\sum_{n=1}^{N} f_{(\Xi_n,\Theta_n)}(k_I + \theta k_R + k_F, \theta)}{\sum_{n=1}^{N} f_{(\Xi_n,\Theta_n)}(k_I + \theta k_R, \theta)}. \tag{E.6}$$

This inequality holds under Assumption 1 as shown above. $\square$

Proof of Lemma 3. (i) By plugging q_S into the objective function in (13), we define $\bar{C}_n(\boldsymbol{k}, \xi, \theta, \gamma) = c_F q_F + \left(\gamma + \frac{b_n}{2}(\xi - k_I - q_R - q_F)\right)(\xi - k_I - q_R - q_F)$, where its derivative wrt q_R and q_F is given

as $-\gamma + b_n(k_I + q_R + q_F - \xi)$ and $c_F - \gamma + b_n(k_I + q_R + q_F - \xi)$, respectively. To determine op-

timal dispatch policy, we consider five cases. Case 1: $\gamma \leq b_n(k_I - \xi)$, $\frac{\partial \bar{C}_n}{\partial q_R} \geq 0$ and $\frac{\partial \bar{C}_n}{\partial q_F} \geq 0$,

hence $q_R^* = q_F^* = 0$. Case 2: $b_n(k_I - \xi) < \gamma \leq b_n(k_I + \theta k_R - \xi)$, $q_R^* = \xi - k_I + \frac{\gamma}{b_n}$ so that

$\frac{\partial \bar{C}_n}{\partial q_R} = 0$, and $q_F^* = 0$ as $\frac{\partial \bar{C}_n}{\partial q_F} \geq 0$. Case 3: $b_n(k_I + \theta k_R - \xi) < \gamma \leq c_F + b_n(k_I + \theta k_R - \xi)$,

$q_R^* = \theta k_R$ so that $\frac{\partial \bar{C}_n}{\partial q_R}$ can be as close to 0 as possible subject to the constraint (14) and $q_F^* = 0$

as $\frac{\partial \bar{C}_n}{\partial q_F}$ is still positive. Case 4: $c_F + b_n(k_I + \theta k_R - \xi) < \gamma \leq c_F + b_n(k_I + \theta k_R + k_F - \xi)$,

similar to the previous case $q_R^* = \theta k_R$, but in this case $q_F^* = \xi - k_I - \theta k_R + \frac{\gamma - c_F}{b_n}$. Case 5:

$c_F + b_n(k_I + \theta k_R + k_F - \xi) < \gamma$, q_R^* remains to be θk_R, and $q_F^* = k_F$ so that $\frac{\partial \bar{C}_n}{\partial q_F}$ can be as

close to 0 as possible subject to the constraint (15). This optimal dispatch policy can be defined

as $q_R^*(\boldsymbol{k}, \xi, \theta, \gamma) = \min\left(\theta k_R, \xi - k_I + \frac{\gamma}{b_n}\right)^{+}$ and $q_F^*(\boldsymbol{k}, \xi, \theta, \gamma) = \min\left(k_F, \xi - k_I - \theta k_R + \frac{\gamma - c_F}{b_n}\right)^{+}$.

(ii) With this optimal dispatch policy, the Hessian of the first stage problem in (17) can be shown

to be positive-definite so that the problem is jointly convex in its arguments. $\square$

Proof of Proposition 4. Let $X_n' = b_n \bar{F}_{\Gamma_n|\Xi_n,\Theta_n}(c_F + b_n(k_I^* + \Theta_n k_R^* + k_F^* - \Xi_n)|\Xi_n, \Theta_n)$ and

$Y_n' = b_n\left[F_{\Gamma_n|\Xi_n,\Theta_n}(c_F + b_n(k_I^* + \Theta_n k_R^* - \Xi_n)|\Xi_n, \Theta_n) - F_{\Gamma_n|\Xi_n,\Theta_n}(b_n(k_I^* + \Theta_n k_R^* - \Xi_n)|\Xi_n, \Theta_n)\right]$.

(i) As in the proof of Proposition 2, we use implicit differentiation to evaluate these derivatives.

$\frac{dk_I^*}{d\alpha_R} = \frac{dk_R^*}{d\alpha_I} = E\left[\sum_{n=1}^{N} X_n'\right] E\left[\sum_{n=1}^{N} \Theta_n Y_n'\right]/H'$, where H' is the determinant of the Hessian (which

is positive). Thus, $\frac{dk_I^*}{d\alpha_R} \geq 0$, indicating that the inflexible and renewable sources are substitutes.

(ii) $\frac{dk_I^*}{d\alpha_F} = \frac{dk_F^*}{d\alpha_I} \geq Z/H$, where $Z = E\left[\sum_{n=1}^N X_n'\right] E\left[\sum_{n=1}^N \Theta_n^2 Y_n'\right] - E\left[\sum_{n=1}^N \Theta_n X_n'\right] E\left[\sum_{n=1}^N \Theta_n Y_n'\right]$.

It can be shown that $Z = \left\{\sum_{n=1}^N b_n (1-q_n) \int_{\xi=0}^\infty \left[\int_{\gamma=c_F+b_n(k_I+k_F-\xi)}^\infty f_{\Gamma_n|\Xi_n,\Theta_n}(\gamma|\xi,0)\,d\gamma\right] f_{\Xi_n|\Theta_n}(\xi|0)\,d\xi\right\}$

$\times \left\{\sum_{n=1}^N b_n q_n \int_{\xi=0}^\infty \left[\int_{\gamma=b_n(k_I+k_R-\xi)}^{c_F+b_n(k_I+k_R-\xi)} f_{\Gamma_n|\Xi_n,\Theta_n}(\gamma|\xi,1)\,d\gamma\right] f_{\Xi_n|\Theta_n}(\xi|1)\,d\xi\right\}$ where we suppress the star

notation and use the Bernoulli intermittency assumption. Hence, Z is positive so that the inflexible and flexible sources are substitutes. (iii) Finally, under Assumption 2(i) and (ii), $\frac{dk_R^*}{d\alpha_F} = $

$\frac{dk_F^*}{d\alpha_R} = -T/H$, where $T = E\left[\sum_{n=1}^N X_n''\right] E\left[\sum_{n=1}^N \Theta_n Y_n''\right] - E\left[\sum_{n=1}^N \Theta_n X_n''\right] E\left[\sum_{n=1}^N Y_n''\right]$. Here,

$X_n'' = b_n \bar{F}_{\Gamma_n}(c_F + b_n(k_I^* + \Theta_n k_R^* + k_F^* - \Xi_n))$ and $Y_n'' = b_n F_{\Gamma_n}(c_F + b_n(k_I^* + \Theta_n k_R^* - \Xi_n))$. Under

the Bernoulli intermittency distribution with $q_n = q$ for all n (Assumption 2(iv)) and for uniform

market price uncertainty (Assumption 2(v)), $T = q(1-q) \sum_{n=1}^N \sum_{m=1}^N b_n b_m \frac{b_n k_R}{U_n - L_n}\left[\frac{c_F}{U_m - L_m}\right]$. Be-

cause T is positive, $\frac{dk_R^*}{d\alpha_F} \leq 0$, hence the renewable and flexible sources are complements. $\quad\square$

Proof of Proposition 5. Under the oversupply penalty, if $\boldsymbol{k} > 0$, the FOC wrt k_I becomes

$$\mathcal{F}(\boldsymbol{k}) = -\sum_{n=1}^N \int_{\theta=0}^1 \left[r_u - (r_u - c_F) F_{(\Xi_n,\Theta_n)}(k_I + \theta k_R + k_F, \theta) - c_F F_{(\Xi_n,\Theta_n)}(k_I + \theta k_R, \theta)\right] f_{\Theta_n}(\theta)\,d\theta + \alpha_I + c_I N$$

$$+ r_o \sum_{n=1}^N F_{\Xi_n}(k_I).$$

The FOCs wrt k_R and k_F remain the same (by letting $r = r_u$) as in the proof of Proposition 2. By

using implicit differentiation, one can show that $\frac{dk_i^*}{dr_o} = \left\{\sum_{n=1}^N F_{\Xi_n}(k_i^*)\right\} \times \frac{dk_i^*}{d\alpha_I}$, for $i \in \{I, R, F\}$. (i)

This part follows from the proof of Proposition 2 that $\frac{dk_I^*}{d\alpha_I} \leq 0$. (ii) Under Assumption 1, as shown

in the proof of Proposition 3, $\frac{dk_F^*}{d\alpha_I}$ and $\frac{dk_R^*}{d\alpha_I}$ are both positive. Hence, $\frac{dk_i^*}{dr_o} \geq 0$ for $i \in \{R, F\}$. $\quad\square$

Proof of Proposition 6. With slight abuse of notation, let $r = r_u$. As shown in the proof of

Proposition 5, if $r_o > 0$, only the FOC wrt k_I changes in the triple sourcing strategy (i..e, $\boldsymbol{k} > 0$).

Hence, it can be shown that $\frac{dk_I^*}{d\alpha_R}$ and $\frac{dk_F^*}{d\alpha_I}$ remain the same as in the proof of Proposition 3, proving

the substitution effects in part (i) and (ii) of this proposition. (iii) By using implicit differentiation,

$\frac{dk_R^*}{d\alpha_F} = h(r_o) = -\left[Tc_F(r - c_F) - r_o \sum_{n=1}^N \int_{\theta=0}^1 f_{(\Xi_n,\Theta_n)}(k_I,\theta)\,d\theta\right]/H$, where T is defined in (E.5)

with $r = r_u$, and $H > 0$ is the determinant of the Hessian evaluated at $\boldsymbol{k}^*$. Note that $h(0) < 0$ if

Assumption 1 holds in the strict sense as the inequality in (E.6) would also be strict in this case

(i.e., $T > 0$). Because $h(r_o)$ is a continuous function and $h(0) < 0$, there exists an $\bar{r} > 0$ such that

$h(r_o) \leq 0$ if $r_o \leq \bar{r}$. Therefore, $h(r_o) = \frac{dk_R^*}{d\alpha_F} \leq 0$ if $r_o \leq \bar{r}$, proving the complementarity. $\quad\square$

Proof of Proposition 7. This proof is omitted as it is similar to the proof of Proposition 1. $\quad\square$

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