



Operations Research

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Technical Note—Managing Inventory for Firms with Trade Credit and Deficit Penalty

Wei Luo, Kevin H. Shang

To cite this article:

Wei Luo, Kevin H. Shang (2019) Technical Note—Managing Inventory for Firms with Trade Credit and Deficit Penalty. Operations Research 67(2):468-478. <https://doi.org/10.1287/opre.2018.1787>

Full terms and conditions of use: <https://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2019, INFORMS

Please scroll down for article—it is on subsequent pages

INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Technical Note—Managing Inventory for Firms with Trade Credit and Deficit Penalty

 Wei Luo,^a Kevin H. Shang^b
^a IESE Business School, University of Navarra, 08034 Barcelona, Spain; ^b Fuqua School of Business, Duke University, Durham, North Carolina 27708

Contact: wluo@iese.edu,  <http://orcid.org/0000-0002-7139-8592> (WL); khshang@duke.edu,  <http://orcid.org/0000-0001-9546-6310> (KHS)

Received: September 25, 2013

Revised: August 17, 2015; November 22, 2016; January 14, 2018

Accepted: May 18, 2018

Published Online in Articles in Advance: March 19, 2019

OR/MS Subject Classifications: inventory/production: policies; finance: working capital

Area of Review: Operations and Supply Chains

<https://doi.org/10.1287/opre.2018.1787>
Copyright: © 2019 INFORMS

Abstract. This paper considers a firm that periodically orders inventory to satisfy demand in a finite horizon. The firm operates under two-level trade credit—that is, it offers trade credit to its customer while receiving one from its supplier. In addition to standard inventory-related costs, the firm also incurs periodic cash-related costs, which include a deficit penalty cost due to cash shortage and an interest gain (negative cost) due to excess cash after inventory payments. The objective is to obtain an inventory policy that maximizes the firm’s working capital at the end of the horizon. We show that this problem is equivalent to one that minimizes the total inventory- and cash-related costs within the horizon. For this general model, we prove that a state-dependent policy is optimal. To facilitate implementation and reveal insights, we consider a simplified model in which a myopic policy is optimal under nondecreasing demand. A numerical study suggests that this myopic policy is an effective heuristic for the original system. The heuristic policy generalizes the classic base-stock policy and resembles practical working capital management under which a firm orders according to its working capital level. The policy parameters have closed-form expressions, which show the impact of demand and cost parameters on the inventory decision. Our study assesses the value of considering financial flows when a firm makes the inventory decision and reveals insights consistent with empirical findings.

Supplemental Material: The online appendices are available at <https://doi.org/10.1287/opre.2018.1787>.

Keywords: trade credit • inventory policy • working capital • default penalty

1. Introduction

Trade credit finance is an important inventory financing tool for firms. Particularly for small or newly established firms, which usually face expensive bank loans, trade credit is the lifeline to their business operations (Klonowski 2014). Managing inventory with trade credit is part of the firm’s working capital management. Working capital refers to the difference between current assets and current liabilities (short-term assets and liabilities with maturities of less than one year). On the balance sheet, current assets include cash, inventory, and accounts receivable (A/R). Current liabilities include accounts payable (A/P) and short-term loans. With trade credit, inventory decisions directly affect working capital levels, as the deferred inventory payment and the delayed sales collection are recorded as A/P and A/R, respectively. Working capital represents liquidity and financial viability for firms. Thus, it is very important to investigate how firms should maximize their working capital when trade credit is present in their business transactions.

We consider a firm in the middle of a supply chain. The firm periodically orders inventory from its supplier to fulfill stochastic demand received from its customer in

a finite horizon. We consider a two-level trade credit financing—that is, the firm offers trade credit to its customer while receiving one from its supplier. The trade credit is a one-part (net term) contract—that is, the payment is due in a certain time period after the invoice is issued. The firm receives sales revenue after a delayed *collection period* following the demand and pays for the ordered inventory after a *payment period* following the delivery of goods. A deficit penalty is charged upon the unfulfilled payment to the supplier. If the firm has access to short-term bank loans, the deficit penalty represents the loan interest rate; otherwise, it represents a payment default penalty charged by the supplier.¹ On the other hand, the firm might have alternative investment functions that yield a positive interest gain from the excess cash after inventory payments.

Most inventory models in the literature assume ample cash supply and do not explicitly consider the interaction between inventory decisions and cash flows. We believe that it is essential to study this connection for the following two reasons. First, during the financial crisis, it is difficult for firms to secure sufficient cash to meet their short-term operations. Consequently, the inventory decision plays a key role in a firm’s liquidity and

operational efficiency. More specifically, a firm's current inventory order directly affects its future cash payment. By ordering too much, it not only incurs a higher holding cost, but also increases the chance of future deficit penalty. On the other hand, ordering too little will increase the chance of stockouts, although a higher return of interest gains is expected. Thus, there are clear tradeoffs between these financial consequences when making an inventory decision. Second, for the traditional inventory problem, there exist very simple, yet powerful, policies (e.g., newsvendor solution, base-stock policy, etc.) that illustrate how inventory decision is affected by the system parameters. In the recent financial crisis, practitioners advocate the importance of interdisciplinary study between operations and accounting/finance. Our goal is to provide a simple policy that illustrates the impact of financial flows on the inventory decision.

We formulate the inventory system with trade credit into a multistate dynamic program that keeps track of inventory level and cash balance, as well as different ages of A/P and A/R within the payment and collection periods, respectively. In addition to standard inventory-related costs, the firm also incurs periodic cash-related costs, which include a deficit penalty cost due to cash shortage and an interest gain (negative cost) due to excess cash after inventory payments. The objective is to obtain an inventory policy that maximizes the firm's expected working capital at the end of the horizon. Note that our objective is not contradicted with maximizing a firm's value, defined in most corporate finance textbooks as the present value of the total dividend payments (e.g., Ross et al. 1996). The dividend payment cycle for most firms is on a quarterly or biannual basis, whereas the inventory order decision is mostly on a daily or weekly basis. Thus, the firm can maximize its working capital so as to better exercise the dividend policy at the end of the planning horizon.

We first show that maximizing the end-of-horizon working capital is equivalent to minimizing the total inventory- and cash-related cost within the horizon. We prove that the optimal policy is a state-dependent, order-up-to policy. From the implementation perspective, it would be difficult to execute a state-dependent policy. Thus, we approximate the model by simplifying the future cash state. We further introduce the notion of *effective working capital* ($= \text{cash} + \text{inventory} + \text{effective accounts receivable} - \text{accounts payable}$) to reduce the corresponding problem into a two-state dynamic program. (The effective accounts receivable is defined in Section 4.) When the payment period is shorter than the collection period, we prove that a myopic policy is optimal for this simplified model under nondecreasing demand. Let c be the unit purchase cost. The myopic policy has a very simple structure with two control parameters (d, S) , $d \leq S$: The firm reviews its effective working capital level and inventory position at the

beginning of each period; if the effective working capital is lower [respectively (resp.), higher] than the deficit threshold level cd (resp., the base-stock level in monetary value cS), the firm places an order to bring its inventory position up to d (resp., S); if the effective working capital level is between cd and cS , the firm orders up to the effective working capital level (in inventory units). When the payment period is longer than the collection period, the firm's working capital level beyond the collection period depends on the future cash inflows, which in turn depend on the future random demands. This complication results in a generalized (d, S) policy, referred to as the $(\mathbf{d}, \mathbf{a}, S)$ policy. Depending on the length of the payment period and the collection period, we then apply either the (d, S) policy or the $(\mathbf{d}, \mathbf{a}, S)$ policy as our heuristic. The heuristic policy resembles practical working capital management under which a firm makes inventory decisions according to the working capital level (Aberdeen Group 2009). A numerical study suggests that the heuristic is effective.

We summarize the main contributions. First, our model and results augment the current scope of operations by incorporating financial considerations. Managers are often hindered from integrating accounts payable/receivable into the inventory policy due to the typical organizational structure of the firm (i.e., the former is a function of a treasurer and the latter an operations manager). These two functions need to be aligned, especially when the firm faces financial market imperfections. We present a model that captures the dynamics between inventory decisions and accounts payable/receivable resulting from trade credit terms and provide a simple heuristic policy. The policy parameters have closed-form expressions, which facilitate classroom teaching and provide a clear intuition for the impact of system parameters on the inventory decision.

Second, our model generalizes the two inventory policies in the literature. When the financial market is perfect—for example, the deficit penalty cost rate equals the interest return rate—the model reduces to the traditional inventory model, and the heuristic policy degenerates into a base-stock policy. With this connection, we can provide a clear economic meaning of cost parameters for the traditional model. For example, the holding cost rate is composed of the physical holding cost rate and the cost of capital is determined by the interest return rate. On the other hand, when the deficit penalty rate is sufficiently large, our policy suggests that the firm order up to the working capital level, which is equivalent to the solution obtained from the cash-constrained model (Bendavid et al. 2017). We believe that a firm without ample internal cash and exercising trade credit for its transactions lies in between these two extremes, and our model reflects this generality. Comparing the heuristic cost with those of these two existing policies also quantifies the value of financial information.

Third, our model reveals insights consistent with empirical findings in the literature. For example, our policy indicates that a firm may choose to bear deficit after payment if its working capital level is low or the backorder cost rate is significantly higher than the deficit penalty rate. This conclusion echoes Cuñat and Garcia-Appendini (2012), in which the authors find that payment defaults are commonly observed in practice, as the default penalty cost is usually small. Our analysis also shows how the lengths of payment period and collection period affect the system cost. We find that the firm can achieve the maximum incremental cost reduction when it keeps an equal length of payment period and collection period. This finding complements an observation that a firm's payment period and collection period are positively correlated (e.g., Fabbri and Klapper 2009).

2. Literature Review

Our paper is related to a few research topics summarized below. We discuss theoretical models, as well as empirical findings, which set the stage for our model assumptions.

The related literature on finite-horizon models can be categorized based on how trade credit is modeled. One category, which is more related to our model, is to explicitly characterize cash-flow dynamics resulted from the trade credit terms. Haley and Higgins (1973) consider the problem of jointly optimizing inventory decision and payment times when demand is deterministic and inventory is financed with trade credit. Schiff and Lieber (1974) consider the problem of optimizing inventory and trade credit policy for a firm with deterministic demand that depends on the credit term and inventory level. Bendavid et al. (2017) study a firm whose replenishment decisions are constrained by the working capital requirement. Their model is similar to ours in that they also consider how inventory replenishment is affected by the payment and the collection periods. However, their model considers independent and identically distributed (i.i.d.) demand and imposes a base-stock policy subject to a working capital constraint. They characterize the dynamics of system variables and obtain the optimal base-stock level via a simulation approach. On the other hand, we provide a simple policy with closed-form expressions for the policy parameters, which show the impact of system parameters on the inventory decision. More importantly, we introduce deficit penalty that not only resembles firm practice, but also generalizes two inventory policies in the literature: base-stock policy and cash-constrained base-stock policy.

Another category is to characterize the impact of trade credit on the inventory holding cost rate. This literature implicitly assumes that cash is always available, so cash dynamics are not explicitly modeled. Beranek (1967)

uses a lot-size model to illustrate how a firm's inventory holding cost should be adjusted according to the firm's actual financial arrangements. Maddah et al. (2004) investigate the effect of permissible delay in payments on ordering policies in a periodic-review (s, S) inventory model with stochastic demand. They develop approximations for the policy parameters. Gupta and Wang (2009) consider a stochastic inventory system where the trade credit term is modeled as a non-decreasing holding-cost rate according to an item's shelf age. Under the assumption that the full payment is made when the item is sold, they prove that a base-stock policy is optimal.

Our model is also related to cash-based inventory problems without trade credit. Chao et al. (2008) consider a self-financed retailer who replenishes inventory in a finite horizon with i.i.d. demand. They consider a lost-sales model, and the available cash forms a hard constraint on the inventory order quantity. They show that a capital-dependent base-stock policy is optimal. Our model considers two-level trade credit and assumes deficit penalty instead of a hard cash constraint. Our heuristic policy also depends on the effective working capital, which is different from what is defined in Chao et al. (2008). As stated, when the deficit penalty is large, our model becomes a cash-constrained model. Li et al. (2013) study a dynamic model in which inventory and financial decisions are made simultaneously in order to maximize the firm's value—the expected present value of dividends minus total capital subscriptions. Luo and Shang (2015) integrate material and cash flows in a supply chain. They characterize the optimal joint inventory and investment policy and investigate the value of cash pooling. Katehakis et al. (2016) study a dynamic model where a firm can finance its inventory with bank loans. On-hand cash earns deposit interest, and short-term debt bears loan interest. Their objective is also to maximize the expected value of the firm's capital at the end of the planning horizon.

The motivation and several key assumptions of our model are based on the empirical findings. Two-level trade credit is commonly seen in practice—that is, firms often provide trade credit to their customer while receiving one from its supplier; see Cuñat and Garcia-Appendini (2012) for an excellent review on the existence of trade credit. Among various types of trade credit contracts, the one-part (net term) is the simplest form. Cuñat (2007) indicates that there is a big portion of firms that use one-part contracts.² Klapper et al. (2012) show direct evidence that most trade credit contracts are of net term. Our paper takes one-part trade credit as a premise and aims to investigate its impact on a firm's inventory policy and the resulting operating cost. Table 1 of Cuñat and Garcia-Appendini (2012) shows the magnitude of late-payment penalties. This supports our deficit penalty assumption. The payment periods in our

model are fixed within the horizon once the firm and its business partners agree upon a net term contract. This is consistent with an empirical finding in Ng et al. (1999), in which the trade credit terms (payment and collection period in our context) may be different across industries, but they are relatively stable within each industry and along time.

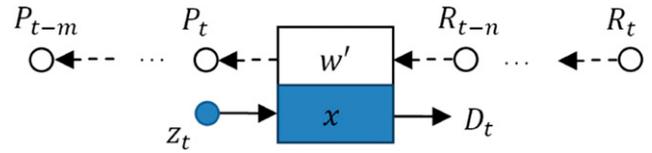
3. The Model

We consider a finite-horizon, periodic-review inventory system where a firm orders from its supplier and sells to its customer. A one-part trade credit contract is employed for transactions with its upstream and downstream partners. That is, the firm pays its supplier after a payment period following the delivery of goods and receives cash from its customer after a collection period following the demand. Transactions based on trade credit affect a firm's accounts payable and accounts receivable, which we now formalize into our model. Because the focus is on cash and inventory dynamics under trade credit, we assume that the shipping lead time is zero for simplicity. Let m be the payment period and n be the collection period. We count the time forward—that is, $t = 0, 1, 2$, etc. The sequence of events is as follows: At the beginning of period t , (1) an inventory order decision is made and a new A/P is generated; (2) a shipment arrives; (3) payment due in this period (corresponding to the inventory ordered in period $t - m$) is made to the supplier; and (4) a deficit penalty cost is incurred in case of insufficient payment (a negative cash level) or an interest return is gained in case of a positive cash level; demand is realized during the period, and a new A/R is generated. Customer payment due in this period (corresponding to the sales in period $t - n$) is collected; at the end of the period, all inventory-related costs are calculated. The objective is to maximize the firm's working capital at the end of the T -period horizon.

In event (4), we assume that a deficit penalty cost is incurred in case of negative cash level after inventory payments. This can be viewed as a scenario in which the firm can raise a short-term loan from banks for the insufficient amount, and the deficit penalty cost rate represents the interest rate of such loan. Alternatively, if the firm has no access to bank loans and only finances its operations through trade credit, the deficit penalty should represent the default penalty charged by the supplier; see the discussion in Endnote 1. The trade credit default cost can be rolled over to the next period if the firm has no cash to pay for the due amount.

Customer demand in period t is modeled as a non-negative random variable D_t with probability density function (p.d.f.) f_t , cumulative distribution function (c.d.f.) F_t , mean μ_t , and variance σ_t^2 . The demand is stochastic and independent between periods. We assume that the unsatisfied demand is fully backlogged.

Figure 1. (Color online) The Base Model with Material Flow and Cash Flow



Define the state and decision variables at the beginning of period t :

- x_t = net inventory level before Event (1);
- y_t = inventory position after Event (2);
- w'_t = net cash level before Event (3);
- $\mathbf{P}_t = (P_{t-m}, \dots, P_{t-1})$: m -dimensional vector of accounts payable;
- $\mathbf{R}_t = (R_{t-n}, \dots, R_{t-1})$: n -dimensional vector of accounts receivable.

Here, P_{t-i} and R_{t-j} denote the A/P and A/R created in period $t - i$ and $t - j$, respectively, for $i = 0, 1, \dots, m$ and $j = 0, 1, \dots, n$. So P_{t-m} and R_{t-n} are the most aged A/P and A/R, whereas P_t and R_t are A/P and A/R created in the current period. Figure 1 shows these system variables in period t with the material and cash flows in solid and dashed arrows, respectively.

Let p be the unit sales price, c be the unit purchase cost, h be the physical holding cost rate, b be the tangible backorder cost rate (e.g., operating costs required to place an expedited order; see Porteus 2002), r be the interest return rate for the positive net cash level after payment, and e be the deficit penalty cost rate for the negative cash level after payment. To avoid any financial arbitrage, we assume that $e > r$. In each period t , the accounts payable created due to the inventory order quantity is $P_t = c(y_t - x_t)$, $y_t \geq x_t$. The accounts receivable generated from the random sales is $R_t = pD_t$. Here, we assume that the customers pay the full amount when the trade credit is due. The interest return on the net cash after payment is $r(w'_t - P_{t-m})^+$, and the deficit penalty cost is $e(P_{t-m} - w'_t)^+$. Define the working capital w_t at the beginning of period t as

$$w_t = cx_t + w'_t - \sum_{i=1}^m P_{t-i} + \sum_{j=1}^n R_{t-j}. \quad (1)$$

Then, the evolution of the system variables are

$$x_{t+1} = y_t - D_t, \quad (2)$$

$$w'_{t+1} = w'_t + R_{t-n} - P_{t-m} - h(y_t - D_t)^+ - b(y_t - D_t)^- - e(P_{t-m} - w'_t)^+ + r(P_{t-m} - w'_t)^-, \quad (3)$$

$$\mathbf{P}_{t+1} = (\mathbf{P}_t^{-1}, c(y_t - x_t)), \quad (4)$$

$$\mathbf{R}_{t+1} = (\mathbf{R}_t^{-1}, pD_t), \quad (5)$$

where we denote \mathbf{P}_t^{-1} as vector \mathbf{P}_t without the first element (same for \mathbf{R}_t^{-1}). Also, define

$$g_t(y_t) = h(y_t - D_t)^+ + b(y_t - D_t)^-, \quad (6)$$

$$v_t(P_{t-m}, w'_t) = e(P_{t-m} - w'_t)^+ - r(P_{t-m} - w'_t)^-. \quad (7)$$

From Equations (1)–(7), after some algebra, the working capital in period $t + 1$ can be expressed as

$$w_{t+1} = w_t + (p - c)D_t - g_t(y_t) - v_t(P_{t-m}, w'_t). \quad (8)$$

We refer to $g_t(y_t)$ as the inventory-related cost and $v_t(w'_t, P_{t-m})$ as the cash-related cost. Equation (8) states that the working capital in period $t + 1$ is equal to the working capital in period t plus net cash flows. With this result, it can be shown that the end-of-horizon working capital equals the initial working capital plus the total net cash flows within the horizon. More specifically,

$$w_{T+1} = w_1 + \sum_{t=1}^T (p - c)D_t - \sum_{t=1}^T g_t(y_t) - \sum_{t=1}^T v_t(P_{t-m}, w'_t).$$

Note that $E[(p - c)D_t]$ is a constant and w_1 is the initial working capital. Thus, maximizing the expected end-of-horizon working capital, $E[w_{T+1}]$, is equivalent to minimizing the expected total inventory-related cost and cash-related cost within the horizon. That is,

$$\min_{y_t \geq x_t, w'_t} \left\{ E \left[\sum_{t=1}^T (g_t(y_t) + v_t(P_{t-m}, w'_t)) \right] \right\}. \quad (9)$$

The problem (9) can be solved from the dynamic program below. Denote $\hat{V}_t(x_t, w'_t, \mathbf{P}_t, \mathbf{R}_t)$ as the minimum expected cost from period t to T among all feasible policies.

$$\hat{V}_t(x_t, w'_t, \mathbf{P}_t, \mathbf{R}_t) = \min_{y_t \geq x_t} \{ (G_t(y_t) + v_t(P_{t-m}, w'_t)) + E \hat{V}_{t+1}(x_{t+1}, w'_{t+1}, \mathbf{P}_{t+1}, \mathbf{R}_{t+1}) \}, \quad (10)$$

where $G_t(y_t) = E[g_t(y_t)]$, and w'_{t+1} follows the dynamics in (3). The terminal condition is

$$\hat{V}_{T+1}(x_{T+1}, w'_{T+1}, \mathbf{P}_{T+1}, \mathbf{R}_{T+1}) = 0.$$

Our model can be viewed as a generalization of the classic inventory model where the firm does not receive or extend trade credit and always has ample cash. Online Appendix A shows how our general model reduces to the classic inventory model as a special case and provides clear economic explanation for the cost parameters therein.

The model formulated in (10) has a state space of $m + n + 2$ dimensions. Let $s_t(x_t, w'_t, \mathbf{P}_t, \mathbf{R}_t)$ denote the optimal solution to this problem. The proposition below shows that the optimal value function \hat{V}_t is jointly convex, which yields a state-dependent optimal stocking level.

Proposition 1. *The optimal policy for the inventory model in (10) is a state-dependent, order-up-to policy, where the target inventory stocking level is $s_t(x_t, w'_t, \mathbf{P}_t, \mathbf{R}_t)$.*

A state-dependent policy is difficult to implement, as solving the dynamic program requires a significant computational effort due to curse of dimensionality. Also, it reveals little insight on how to manage the system. Our idea is to present a simplified model that eliminates the curse of dimensionality and show that a simple policy is optimal to this simplified model. This optimal policy will then be used as the heuristic policy for the original system.

3.1. Simplified Model

For the problem in (9), the order decision y_t affects the inventory holding and backorder cost $g_t(y_t)$ in period t and the cash-related cost $v_{t+m}(w'_{t+m}, P_t)$ in period $t + m$. Although this problem has a similar structure as the classic inventory model with lead time L (where the order decision in period t affects the ordering cost in period t and the inventory-related cost in period $t + L$), the cash level w'_{t+m} is jointly determined by the future inventory decisions y_{t+m-i} , $i = 1, \dots, m$. More specifically, applying Equation (3) recursively, we have

$$w'_{t+m} = w'_t + \sum_{i=1}^m R_{t+m-n-i} - \sum_{i=1}^m P_{t-i} - \sum_{i=1}^m g_{t+m-i}(y_{t+m-i}) + \sum_{i=1}^m v_{t+m-i}(P_{t-i}, w'_{t+m-i}).$$

As shown, w'_{t+m} is jointly determined by y_{t+m-i} through all inventory- and cash-related costs (note that w'_{t+m-i} depends on y_{t+m-i}). This dependence leads to curse of dimensionality.

In order to resolve this issue, we simplify the model by omitting these inventory- and cash-related costs in the cash state dynamics (although we still include these costs in the objective function).

Assumption 1. *The inventory and cash-related costs are omitted in the cash dynamics.*

Under Assumption 1, the cash transition in (3) becomes

$$w'_{t+1} = w'_t + R_{t-n} - P_{t-m},$$

and we can resolve the curse of dimensionality issue in the exact system. We use the above state transition to solve the problem in (10) and refer to the new system as the *simplified model*.

This simplification is supported by the following logic: Under a well-managed system, the inventory- and cash-related costs are minimized. As a result, the gap of the working capital levels between the exact system and the simplified system should be minimal. This will lead to similar behaviors of these two systems. To further

strengthen this logic, we have conducted a simulation study to compute the working capital difference between the exact system and the simplified system when our proposed heuristic policy is implemented. With a total number of 2,187 instances generated from the test bed (see Online Appendix C), the average percentage difference between the two end-of-horizon working capital levels is 0.05%. These results verify that the impact of the omission is minimal.

We remark that omitting the inventory- and cash-related costs in the dynamics does not deviate too much from practice. Specifically, although the inventory-related cost (i.e., the sum of holding and backorder costs) may occur in each period, they often do not realize in a firm’s operational account until the end of a planning horizon. This observation is consistent with the problem formulation in our simplified model: The inventory-related cost is not included in periodic cash dynamics, but is accounted for in the objective function, which minimizes the total cost during the planning horizon (or, equivalently, maximizes the end-of-horizon working capital).

4. Heuristic Policies

Under the simplified model described in Section 3, the inventory decision y_t affects the payment as well as the cash-related cost in period $t + m$. Depending on the lengths of the collection period and the payment period, the problem in (10) can be reformulated into two different dynamic programs. When $m \leq n$, the A/R pipeline is longer than the A/P pipeline, in which case the cash balance in period $t + m$ is totally known; we present the solutions in Section 4.1. When $m > n$, the cash balance in period $t + m$ becomes a random variable, and the corresponding analysis becomes much more involved; we discuss the main results in Section 4.2 and leave the detailed analysis to Online Appendix B.

4.1. The System with a Longer Collection Period

For the system with a longer collection period—that is, $m \leq n$ —the inventory decision y_t affects the cash-related cost in period $t + m$. More specifically, under Assumption 1,

$$\begin{aligned} w'_{t+m} &= w'_t + \sum_{i=1}^m R_{t+m-n-i} - \sum_{i=1}^m P_{t-i} \\ &= \left(w'_t + \sum_{j=1}^n R_{t-j} - \sum_{i=1}^m P_{t-i} \right) - \sum_{k=1}^{n-m} R_{t-k} \\ &= w_t - \left(\sum_{k=1}^{n-m} R_{t-k} \right) - cx_t. \end{aligned} \quad (11)$$

The cash-related cost in period $t + m$ is determined by $(P_t - w'_{t+m})$, which is

$$P_t - w'_{t+m} = cy_t - \left(w_t - \sum_{k=1}^{n-m} R_{t-k} \right). \quad (12)$$

Notice that $\sum_{k=1}^{n-m} R_{t-k}$ is known in period t . Let us define the *effective* working capital as follows:

$$\underline{w}_t = w_t - \sum_{k=1}^{n-m} R_{t-k}, \quad (13)$$

which is equal to the working capital in period t , excluding the known accounts receivable in periods $t - n + m, \dots, t - 1$.

The optimal solution to the simplified system can be obtained from the following dynamic program: $V_{T+1}(x, \underline{w}) = 0$, and

$$V_t(x, \underline{w}) = \min_{y \geq x} \{ G_t(y) + v_t(cy, \underline{w}) + E[V_{t+1}(y - D_t, \underline{w} + R_{t-n+m} - cD_t)] \}. \quad (14)$$

For notational simplicity, we do not explicitly include the known accounts receivables $R_{t-n+m}, \dots, R_{t-1}$ in the state space. For the special case of $m = n$, the effective working capital dynamics become $\underline{w}_{t+1} = \underline{w}_t + (p - c)D_t$. Without confusion, we omit the time index t in the state variables in the sequel. We can show that a state-dependent policy is optimal for the above dynamic program.

Proposition 2. (1) $V_t(x, \underline{w})$ is jointly convex in x and \underline{w} ; (2) a state-dependent base stock policy $s_t(x, \underline{w})$ is optimal; that is, order up to $s_t(x, \underline{w})$ if $x \leq s_t(x, \underline{w})$ and do not order otherwise.

Proposition 2 shows that by introducing the concept of effective working capital and employing Assumption 1, we can reduce the original problem from $(m + n + 2)$ states to two states. This makes the computation possible. However, from a perspective of implementation and revealing insights, a state-dependent policy is not ideal. Below, we shall introduce a simple and implementable policy.

We first solve the single-period problem in (14) and obtain the corresponding myopic policy, referred to as the (d, S) policy. We then show the (d, S) policy is indeed optimal for the finite horizon problem in (14) when the demand is stochastically nondecreasing. As we shall see in Online Appendix C, the policy remains very effective for the nonstationary demand case in our numerical study.

Without considering the constraint, the single-period problem in (14) can be written as

$$v_t(\underline{w}) = \min_y \{ G_t(y) + e(cy - \underline{w})^+ - r(cy - \underline{w})^- \}, \quad (15)$$

where $G_t(y_t) = E[g_t(y_t)]$. To facilitate our discussion, we first define the control parameters:

$$d_t = \left\{ y : \frac{\partial}{\partial y} G_t(y) = -ec \right\}; \quad S_t = \left\{ y : \frac{\partial}{\partial y} G_t(y) = -rc \right\}. \quad (16)$$

Or, equivalently,

$$F_t(d_t) = \frac{b - ec}{b + h}; \quad F_t(S_t) = \frac{b - rc}{b + h}. \quad (17)$$

To solve the problem in (15), we consider three cases; see Figure 2(a).

Case 1. When $\underline{w} \leq cd_t$, the system’s effective working capital is lower than the deficit threshold cd_t . In this case, the firm has an incentive to order up to d_t as the marginal backorder cost, and interest return outweighs the marginal holding and deficit penalty cost. Thus, we have $v_t(\underline{w}) = L_t(\underline{w}) = G_t(d_t) - e(\underline{w} - cd_t)$.

Case 2. When $cd_t < \underline{w} \leq cS_t$, the system is constrained by the effective working capital. It is optimal to order up to \underline{w}/c as ordering either less or more will lead to a higher cost than $G_t(\underline{w}/c)$. Thus, $v_t(\underline{w}) = G_t(\underline{w}/c)$.

Case 3. When $cS_t < \underline{w}$, the system has ample effective working capital and orders up to the target base-stock S_t . In this case, there is extra cash left after ordering, which yields an interest return of $r(\underline{w} - cS_t)^+$. In this case, $v_t(\underline{w}) = R_t(\underline{w}) = G_t(S_t) - r(\underline{w} - cS_t)$.

As a result, Equation (15) becomes

$$v_t(\underline{w}) = \begin{cases} L_t(\underline{w}), & \text{if } \underline{w} \leq cd_t \\ G_t(\underline{w}/c), & \text{if } cd_t < \underline{w} \leq cS_t \\ R_t(\underline{w}), & \text{if } cS_t < \underline{w} \end{cases} \quad (18)$$

Denote y_t^* as the resulting optimal order-up-to level under the (d, S) policy; that is,

$$y_t^*(\underline{w}) = (d_t \vee \underline{w}/c) \wedge S_t. \quad (19)$$

We define the *band* where the initial inventory level x is less than or equal to $y_t^*(\underline{w})$ as follows:

$$\mathfrak{B}_t = \{(x, \underline{w}) \in \mathfrak{R}^2 \mid x \leq y_t^*(\underline{w})\}. \quad (20)$$

Figure 2(b) depicts the piecewise linear function $y_t^*(\underline{w})$. By definition, the band \mathfrak{B} covers the area below $y_t^*(\underline{w})$ on the x - \underline{w} plain. If $x \leq y_t^*(\underline{w})$, the (d, S) policy is optimal for the myopic problem. We summarize this result below.

Proposition 3. *The (d, S) policy is optimal for the myopic problem in (15). The firm monitors its inventory level x and working capital \underline{w} at the beginning of each period. If $\underline{w}/c \leq d_t$, the firm orders up to d_t ; if $d_t < \underline{w}/c \leq S_t$, the firm uses up all cash and orders inventory up to \underline{w}/c ; if $\underline{w}/c > S_t$, the firm orders inventory up to S_t .*

We next show that the (d, S) policy is indeed optimal for the finite-horizon problem when $n \geq m$.

Proposition 4. *If D_t is stochastically increasing in t ,*

(a) *The control parameters d_t and S_t are nondecreasing in t and $d_t \leq S_t$ for all t ;*

(b) *$V_t(x, \underline{w}) = W_t(\underline{w})$ for all t and $(x, \underline{w}) \in \mathfrak{B}_t$, where*

$$W_t(\underline{w}) = v_t(\underline{w}) + EW_{t+1}(\underline{w} + R_{t-n+m} - cD_t),$$

and $W_{T+1}(\underline{w}) = 0$; $W_t(\underline{w})$ is convex in \underline{w} ; and

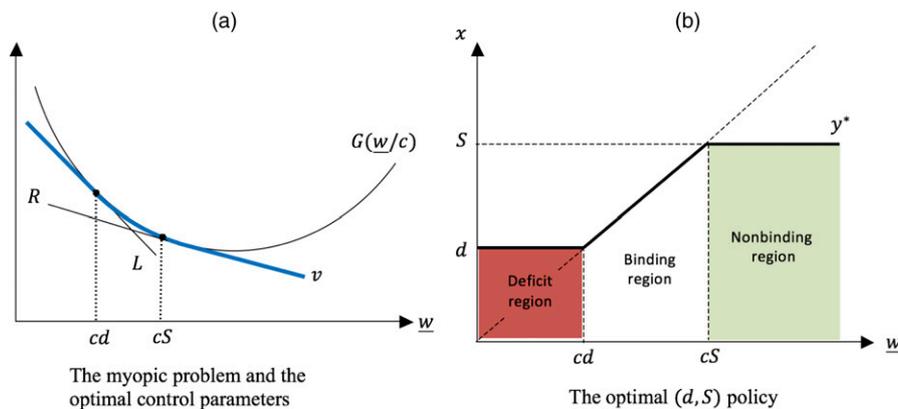
(c) *the (d, S) policy is optimal for the dynamic program in (14).*

Proposition 4(a) implies that if the initial state (x, \underline{w}) falls in the band defined in (20), the system states at the beginning of each period will remain in the band under the (d, S) policy. This property ensures the optimality of the myopic policy, which is similar to the classic inventory model.

The (d, S) policy reveals interesting insights on managing working capital. The firm will have a chance to have cash deficit when d_t is positive. From (17), this scenario happens when $b \geq ec$. In general, when the deficit penalty is sufficiently large, the firm will be less likely to have negative cash, so the resulting model is similar to the cash-constrained model (i.e., cash becomes a hard constraint that restricts the inventory decision) studied by Bendavid et al. (2017). On the other hand, when the financial market is perfect, that is, $e = r$, d_t and S_t are equal. In this case, the firm can always borrow sufficient cash for operations and cash is no longer a concern. Thus, the (d, S) policy is degenerated into a classic base-stock policy.

To formally characterize the firm’s order strategy under deficit risk (induced by raising bank loans or

Figure 2. (Color online) The Optimal Solution of the Simplified Model



defaulting on the supplier), we define the *deficit quantity* as $u^*(\underline{w}) = (cy^*(\underline{w}) - \underline{w})^+$. Figure 2(b) implies that $u^*(\underline{w})$ is decreasing in \underline{w} , meaning that the firm will have less deficit if there is more working capital. This behavior echoes the empirical findings that the operational decisions of smaller firms are more aggressive and thus induce higher deficit risks.

4.2. The System with a Longer Payment Period

For the system with a longer payment period—that is, $m > n$ —we can derive similar cash and working-capital dynamics as those in (11) and (12). More specifically,

$$w'_{t+m} = w'_t + \sum_{j=1}^n R_{t-j} - \sum_{i=1}^m P_{t-i} + \sum_{k=1}^{m-n} R_{t+k-1},$$

$$P_t - w'_{t+m} = cy_t - \left(w_t + \sum_{k=1}^{m-n} R_{t+k-1} \right).$$

The resulting dynamic program is $V_{T+1}(x, w) = 0$, and

$$V_t(x, w) = \min_{y \geq x} \left\{ G_t(y) + \mathbb{E} \left[e \left(cy - \left(w + \sum_{k=1}^{m-n} R_{t+k-1} \right) \right)^+ - r \left(cy - \left(w + \sum_{k=1}^{m-n} R_{t+k-1} \right) \right)^- + V_{t+1}(y - D_t, w + (p - c)D_t) \right] \right\}. \quad (21)$$

Note that $\sum_{k=1}^{m-n} R_{t+k-1} = \sum_{k=1}^{m-n} pD_{t+k-1}$ is a random variable, which is unknown in period t . For ease of exposition, define $m' = m - n$ and $D_t^{m'} = \sum_{k=1}^{m'} D_{t+k-1}$. In addition, let $F^{m'}$, $f^{m'}$, $\mu^{m'}$, and $(\sigma^{m'})^2$ be the c.d.f., the p.d.f., mean, and variance of the random variable of $D^{m'}$, respectively. Moreover, denote $\bar{F}^{m'}$ and $\hat{F}^{m'}$ as the complementary c.d.f. (c.c.d.f.) and the loss function of random variable $D^{m'}$. That is, $\hat{F}^{m'}(x) = \int_x^\infty \bar{F}^{m'}(y) dy$. Equation (21) becomes

$$V_t(x, w) = \min_{y \geq x} \left\{ G_t(y) + \mathbb{E} \left[e \left(cy - (w + pD_t^{m'}) \right)^+ - r \left(cy - (w + pD_t^{m'}) \right)^- \right] + \mathbb{E} V_{t+1}(y - D_t, w + (p - c)D_t) \right\}. \quad (22)$$

Similar to the model with a longer collection period, we can prove that a state-dependent order-up-to policy is optimal.

Proposition 5. (1) $V_t(x, w)$ is jointly convex in x and w . (2) Let $s_t(x, w)$ be the optimal solution. The optimal policy is to order up to $s_t(x, w)$ if $x \leq s_t(x, w)$ and not to order otherwise.

Again, one can compute the optimal policy by introducing the notion of working capital for this simplified model. With the same intention of making the system

more transparent and manageable, we develop two heuristic policies based on two different linear approximation methods: (d, S) policy from the two-piece approximate model and $(\mathbf{d}, \mathbf{a}, S)$ policy from the three-piece approximate model. We refer the reader to Online Appendix B for detailed analysis. Below, we summarize the main ideas.

We define the *effective working capital* for this model as

$$\bar{w} = w + p\mu^{m'},$$

where the second term is the *expected A/R* within m' periods. We first describe the optimal solution to the two-piece approximate model. Unlike the model with a longer collection period, the cash-related cost is convex. We replace the convex cost function with a two-piece linear function, and the resulting problem shares the same structure as before. Therefore, the (d, S) policy is optimal and operated exactly the same as in Section 4.1, except that the system monitors \bar{w} instead of \underline{w} .

$$y^*(\bar{w}) = (d \vee (\bar{w}/c)) \wedge S. \quad (23)$$

To reflect the impact of demand variability on the cash-related cost, we next introduce another approximate model. By replacing the same convex cost function with a three-piece linear function, we are able to develop the $(\mathbf{d}, \mathbf{a}, S)$ policy consisting of five control parameters $(\mathbf{d}, \mathbf{a}, S)$, where $\mathbf{d} = (d, \bar{d})$ and $\mathbf{a} = (a', a'')$. The firm implements a base-stock policy with the optimal base-stock level depending on the effective working capital \bar{w} . In particular, if the firm's expected working capital \bar{w} is lower than \bar{d} , the expected deficit threshold, the firm will order more than its expected working capital, resulting in bank loans or payment defaults. On the other hand, if $\bar{w} > \bar{d}$, the firm will order less than its expected working-capital level and hold extra cash on expectation. Notice that the over-order (under-order) deviation amount depends on a'' (a'), which is proportional to the variance of the aggregated demand. More specifically, let $\bar{y}^*(\bar{w})$ be the optimal order-up-to level. Then, we have,³

$$\bar{y}^*(\bar{w}) = \begin{cases} d, & \text{if } \bar{w} \leq cd - a'' \\ (\bar{w} + a'')/c, & \text{if } cd - a'' < \bar{w} \leq c\bar{d} - a'' \\ \bar{d}, & \text{if } c\bar{d} - a'' < \bar{w} \leq c\bar{d} + a' \\ (\bar{w} - a')/c, & \text{if } c\bar{d} + a' < \bar{w} \leq cS + a' \\ S, & \text{if } cS + a' < \bar{w} \end{cases}. \quad (24)$$

The (d, S) and $(\mathbf{d}, \mathbf{a}, S)$ policies can serve as heuristics for the model with a longer payment period. It is conceivable that the $(\mathbf{d}, \mathbf{a}, S)$ policy works better than the (d, S) policy, although the latter involves less control parameters, and thus is easier to implement. The performance gap between these two heuristic policies

gets bigger when aggregated demand is more volatile. In practice, the (d, S) policy could serve as a simple alternative if the demand is less variable.

5. Numerical Study

In Online Appendix C, we examine the effectiveness of the (d, S) policy and the $(\mathbf{d}, \mathbf{a}, S)$ policy by developing a lower bound to the optimal cost of the exact system.⁴ An intensive test bed study is provided. In Section 5.1, we compare the (d, S) policy with two known inventory-control policies in the literature. We illustrate the importance of collaboration between operations and accounting/finance departments when making the inventory decision. In Section 5.2, we discuss the impact of trade credit periods on the system total cost.

5.1. Value of the Financial Information

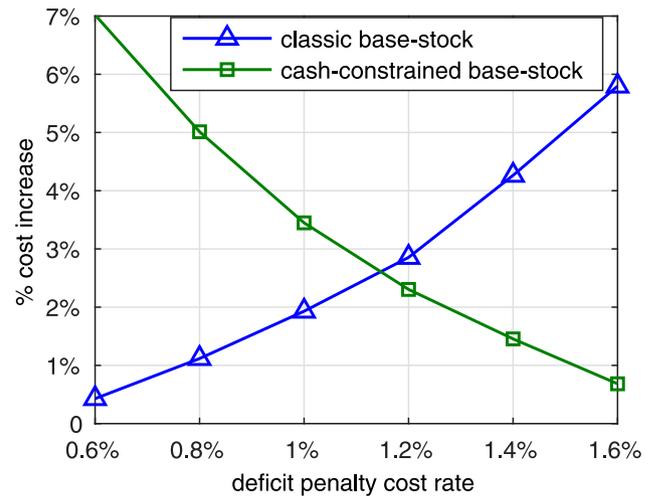
Recall that the optimal (d, S) policy reduces to the classic base-stock policy when $e = r$ and the cash-constrained base-stock policy when e is large. In this subsection, we assess the value of the (d, S) policy over these two policies through a numerical study. Notice that the cost difference between the (d, S) policy and the classic base-stock policy can be viewed as the value of considering financial flows when making the inventory decision.

We focus on the case of $m = n = 1$ and compute the percentage cost increase of each policy over the optimal (d, S) policy under different deficit penalty costs. (Note that this percentage increases in m or n .) Demand D_t is normally distributed with the first period mean $\mu_1 = 10$, and μ_t increasing at a rate of 5% per period. The time horizon is $T = 10$ periods. We set parameter $c = 1, p = 1.05, \sigma_t = 3, h = 3\%$, and $b = 9\%$ $r = 0.1\%$. We vary the deficit penalty cost rate from 0.6% to 1.6%. Online Appendix C explains the construction of the test bed regarding the choice of parameter values.

As shown in Figure 3, the performance of the classic base-stock policy is fairly effective when the deficit penalty cost e is close to the interest return rate r , but becomes less effective when e increases. Recall from (17) that when e approaches r , the deficit threshold d and the base-stock level S come closer to each other. Thus, the (d, S) policy behaves similarly to the base-stock policy. On the contrary, when e is large, d becomes smaller, indicating that the system should order up to either d or \underline{w} instead of S under the base-stock policy. This result shows the importance of interdepartmental collaboration for firms that are typically falling short of cash and subject to high deficit penalty costs.

Conversely, the performance of cash-constrained base-stock policy is quite effective under a large e , but becomes less effective when e decreases. This is because when e is large, d becomes small. The firm would order up to the system working capital \underline{w} more frequently, making the (d, S) policy similar to the cash-constrained policy.

Figure 3. (Color online) Impact of Deficit Penalty on the Value of the Optimal (d, S) Policy



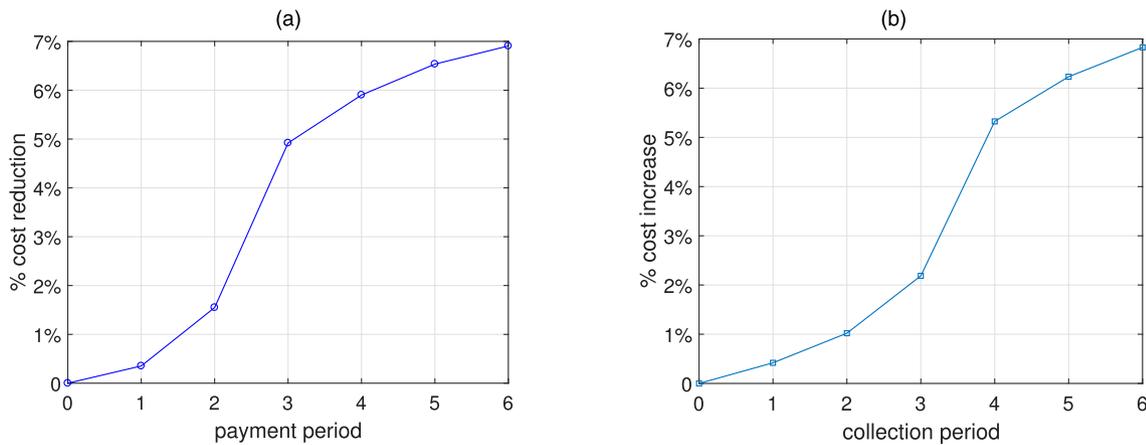
5.2. Impact of Trade Credit Periods

The payment and collection periods jointly affect the *cash conversion cycle* (CCC), which is defined as $CCC = \text{Inventory conversion period} + \text{Collection period} - \text{Payment period}$. Firms typically aim to reduce their cash conversion cycle by extending the payment period and shortening the collection period. However, each trade credit term is often associated with a distinct sales price. For example, although the firm may benefit from a longer payment period, the supplier will usually quote a higher unit wholesale price to compensate for the postponed cash inflow; similarly, while suffering a longer collection period, the firm could also compensate itself by increasing the unit selling price. To fully understand this tradeoff, we need to analyze the cost implications of extending trade credit periods. Particularly in this subsection, we conduct a numerical study to illustrate the impact of extending a firm's payment (collection) period on its total cost reduction (increase). The results could be used as a decision support tool when negotiating trade credit terms with supply chain partners.

We first focus on the upstream trade credit and compute the percentage cost reduction achieved by increasing the payment period m from 0 to 6 while fixing the collection period $n = 3$. All else being equal, extending the payment period m will reduce the total system cost due to enhanced cash flow. To quantify this cost reduction, we keep the unit purchasing cost c unchanged. In the numerical study, we use the (d, S) policy to compute the system cost for the model with longer collection period and the $(\mathbf{d}, \mathbf{a}, S)$ policy for the model with longer payment period. We set $h = 9\%$, $b = 12\%$, $e = 0.4\%$, $r = 0.1\%$, and $\sigma_t = 2$. The rest of the parameters are the same as in Section 5.1.

Figure 4(a) plots the percentage cost reduction curves when extending the payment period m from 0 to 6 while keeping the collection period $n = 3$. For example,

Figure 4. (Color online) Impact of Extending Trade Credit Periods on the Total System Cost



Cost reduction by extending m from 0 to 6 ($n = 3$)

Cost increase by extending n from 0 to 6 ($m = 3$)

a firm with a replenish cycle of 10 days that extends a 30-day trade credit to customers will fit in our model with $n = 3$. As shown in the figure, the total system cost will reduce by 5% when the firm extends payment period m from 0 (pay on order) to 3 (30-day trade credit), and an additional 2% when further extending m from 3 to 6 (60-day trade credit). This suggests that (1) the total system cost decreases with the length of the payment period; and (2) the percentage cost reduction is nonlinear. More specifically, the firm has a stronger incentive to bring the payment period to match with the collection period than further extending beyond it.⁵ To explain this, note that by maintaining a balanced trade credit term ($m = n$), the firm has complete and certain information of its cash flow within the credit periods, which facilitates better management of inventory; when demand is increasing, this synchronization of order payment with revenue collection could bring further benefit by making cash inflow “just in time” for inventory procurement.

Next, we focus on the downstream trade credit and compute the percentage cost increase by extending the collection period n from 0 to 6 while fixing the payment period $m = 3$. Intuitively, delayed revenue collection hurts the firm’s cash flow and thus increases the total system cost. We quantify this cost increase using the same set of parameters. As shown in Figure 4(b), the previously reviewed insight still holds: The firm has a stronger cost incentive to move *toward* a balanced trade credit than *away from* it.⁶ Thus, assuming that the firm follows the optimal policies derived in Sections 4.1 and 4.2, we shall expect that the firm’s upstream and downstream credit periods are positively correlated, as suggested by Guedes and Mateus (2009).

6. Conclusion

This paper studies the impact of two-level trade credit on a firm’s inventory decision. We introduce a notion of

effective working capital that simplifies the computation and characterizes the effective heuristic policies. The resulting policies resemble the business practice of working-capital management in that firms review their working-capital status when making inventory decisions. Our study reveals insights on the importance of collaboration between operations and accounting/finance departments within a firm. In addition, our model generalizes the classic base-stock and the cash-constrained models. We believe that this generality reflects the real-world practice. The policy control parameters have a closed-form expression, which facilitates interdisciplinary teaching for students and practitioners. Finally, we analyze the cost impact of trade credit terms and show that firms have a stronger cost incentive to move toward a balanced trade credit than away from it.

A possible future work is to incorporate demand forecasting in the current model. Aviv (2007) demonstrates the benefit of sharing the forecast demand information with the upstream supplier. In this joint material and cash-flow model, the demand forecast information will be translated into cash-flow information. It will be of interest to investigate the value of demand and cash-flow information for firms.

Acknowledgments

The authors thank the editors and anonymous reviewers for their valuable suggestions to improve the paper. They also thank Vlad Babich, Victor Martinez de Albeniz, Fehmi Tanrisever, the 2013 Supply Chain Finance Symposium participants, and the interface of Finance, Operations, and Risk Management Special Interest Group 2013 Conference participants for their useful comments.

Endnotes

¹As suggested by the 1998 National Survey of Small Business Finances (NSSBF) data, a big portion of the firms declared that they had made some payments to their suppliers after the due date of the trade

credit. These post-due-date payments, referred to as payment defaults, often incur monetary penalties for the buyers.

² According to the 1998 NSSBF survey, 49% of the trade credit contracts are one-part.

³ Online Appendix B explains how these control parameters are obtained.

⁴ Because of the curse of dimensionality, it is computationally infeasible to compute the exact optimal cost. Thus, we shall compare the heuristic cost with the lower bound cost.

⁵ As shown in the figure, when $n = 3$, the firm achieves much higher cost reduction by extending m from 2 to 3 than from 3 to 4.

⁶ As shown in the figure, when $m = 3$, the firm incurs much lower cost increase by extending n from 2 to 3 than from 3 to 4.

References

- Aberdeen Group (2009) Inventory management: 3 keys to freeing working capital. Benchmark report, Aberdeen Group, Boston.
- Aviv Y (2007) On the benefits of collaborative forecasting partnerships between retailers and manufacturers. *Management Sci.* 53(5):777–794.
- Bendavid I, Herer Y, Yücesan E (2017) Inventory management under working capital constraints. *J. Simulation* 11(1):62–74.
- Beranek W (1967) Financial implications of lot-size inventory models. *Management Sci.* 13(8):B401–B408.
- Chao X, Chen J, Wang S (2008) Dynamic inventory management with cash flow constraints. *Naval Res. Logist.* 55(8):758–768.
- Cuñat V (2007) Trade credit: Suppliers as debt collectors and insurance providers. *Rev. Financial Stud.* 20(2):491–527.
- Cuñat V, Garcia-Appendini E (2012) Trade credit and its role in entrepreneurial finance. Cumming D, ed. *Oxford Handbook of Entrepreneurial Finance* (Oxford University Press, New York), 526–557.
- Fabbri D, Klapper L (2009) Trade credit and the supply chain. Working paper, University of Amsterdam, Amsterdam.
- Gallagher T, Andrew J (2007) Working capital policy. *Financial Management: Principles and Practice* (Freeload Press, St. Paul, MN), 478–499.
- Guedes J, Mateus C (2009) Trade credit linkages along a supply chain: evidence for the Italian textile sector. Working paper, Universidade Catolica Portuguesa, Lisbon, Portugal.
- Gupta D, Wang L (2009) A stochastic inventory model with trade credit. *Manufacturing Service Oper. Management* 11(1):4–18.
- Haley W, Higgins R (1973) Inventory policy and trade credit financing. *Management Sci.* 20(4):464–471.
- Heyman D, Sobel M (1984) *Stochastic Models in Operations Research*, vol. 2 (McGraw-Hill, New York).
- International Trade Centre (2009) *How to Access Trade Credit: A Guide to Explore SMEs* (International Trade Centre, Geneva).
- Katehakis MN, Melamed B, Shi J (2016) Cash-flow based dynamic inventory management. *Production Oper. Management* 25(9):1558–1575.
- Klapper L, Laeven L, Rajan R (2012) Trade credit contract. *Rev. Financial Stud.* 25(3):838–867.
- Klonowski D (2014) *Strategic Entrepreneurial Finance: From Value Creation to Realization* (Routledge, New York).
- Li L, Shubik M, Sobel M (2013) Control of dividends, capital subscriptions, and physical inventories. *Management Sci.* 59(5):1107–1124.
- Long M, Malitz I, Ravid S (1993) Trade credit, quality guarantees, and product marketability. *Financial Management* 22(4):117–127.
- Luo W, Shang K (2015) Joint inventory and cash management for multi-divisional supply chains. *Oper. Res.* 63(5):1098–1116.
- Maddah B, Jaber M, Abboud N (2004) Periodic review (s, S) inventory model with permissible delay in payments. *J. Oper. Res. Soc.* 55(2):147–159.
- Modigliani F, Miller M (1958) The cost of capital, corporation finance and the theory of investment. *Amer. Econom. Rev.* 48(3):261–297.
- Ng C, Smith J, Smith R (1999) Evidence on the determinants of credit terms used in interfirm trade. *J. Finance* 54(3):1109–1129.
- Pass C, Pike R (1984) An overview of working capital management and corporate financing. *Managerial Finance* 10(3):1–11.
- Porteus E (2002) *Foundations of Stochastic Inventory Theory* (Stanford University Press, Stanford, CA).
- Ross S, Westerfield R, Jaffe J (1996) *Corporate Finance*, 4th ed. (McGraw-Hill/Irwin, New York).
- Schiff M, Lieber Z (1974) A model for the integration of credit and inventory management. *J. Finance* 29(1):133–140.

Kevin Shang is a professor at the Fuqua School of Business, Duke University. His research mainly focuses on developing simple and effective inventory policies for supply chains. He also conducts research related to the interface of operations and finance and renewable energy systems.

Wei Luo is an assistant professor at IESE Business School. He holds a PhD in operations management from the Fuqua School of Business, Duke University. His research interests include supply chain management, inventory planning and control, and the interface of operations and finance. His recent research focuses on the integration of inventory and cash management, aiming to study the impact of financial flows on operational decisions and supply chain efficiency.