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Cash Beer Game

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ABSTRACT

This article introduces a new online simulation game called *Cash Beer Game*, which is an augmented version of the standard Beer Game by including cash flows. In addition to the inventory ordering and shipping activities, each player pays cash for the ordered inventory to her upstream partner and receives cash from her downstream partner. The goal of this game is to explain the interactions between material, information, and financial flows in a supply chain and help students understand the impact of financial flows on the inventory decision. The resulting bullwhip effect can be compared between teams and with that of the standard Beer Game for the same team.

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1

Motivation

The goal of supply chain management is to match supply with demand effectively by coordinating the activities of multiple firms involved in the production, distribution, and sales of a physical good. The performance of a supply chain depends on how well the material flows, information flows, and financial flows within the supply chain are coordinated. The intertwined relationships between these flows have made this coordination process challenging in practice. To demonstrate the challenge of coordination between the material and information flows to students, the “Beer Game" is often employed in operations management (OM) courses. While a supply chain includes financial flows, they are seldom discussed in the OM curriculum. The 2008 financial crisis has demonstrated the importance of incorporating financial flows into operations decisions – many supply chains were disrupted because upstream firms failed to maintain their normal operations due to financial illiquidity. To illustrate the importance of considering financial flows when making inventory decisions, we augment the Beer Game by incorporating cash flows. We refer to this revised game as *Cash Beer Game*.

Section 2 provides an inventory model with cash flows that sets the stage for the formulation of Cash Beer Game. More specifically, we provide a condition under which the inventory decision and cash flows can be decoupled. This result leads to the standard inventory model in the literature as well as the Beer Game. On the other hand, when this condition does not hold, the cash flow does influence the inventory decision. This motivates the formulation of the Cash Beer Game. Section 3 discusses the pedagogical goals, illustrates the rules and settings of the game, and depicts how the game is delivered in a lecture. Section 4 discusses lessons from past teaching.

2

An Inventory Model with Cash Flows

Consider a firm facing nonstationary random demand in a finite horizon. At the beginning of each period, an order is placed to meet the uncertain demand. The objective is to maximize the expected net worth (equity) at the end of the horizon. For simplicity, let us assume the firm has no investment activities other than investing in inventory and in the capital market with interest returns based on the cash it accumulates. Also, we do not consider long-term assets and liabilities. Thus, the objective is equivalent to maximizing the expected working capital at the end of the horizon. Different objectives have been proposed in the textbooks. For example, Baye and Frince (2014) define that the value of a firm is the present value of the firm's current and future profits. As we shall see later, this objective is equivalent to maximizing the expected terminal working capital we assume here. In some corporate finance textbooks, the value of a firm is defined as the present value of the total dividend payments. Notice that the dividend payment cycle for most firms is on a quarterly or bi-annual basis, whereas the inventory order decision is mostly on a daily or weekly basis. Thus, our objective of maximizing the net worth of the firm does not contradict the latter perspective as the firm can maximize its net worth (equity) for better utilizing its dividend policy (i.e., either paying dividends with cash or keeping cash as retained earnings) at the end of the planning horizon.

Without loss of generality, we assume that the order lead time is zero. Define the following parameters for the system.

$$
\begin{aligned}
h_p &= \text{physical holding cost rate (\$/unit/period);} \\
b_p &= \text{physical backorder cost rate (\$/unit/period);} \\
r &= \text{interest return rate for the cash;} \\
d &= \text{borrowing rate for the debt, where } d \geq r; \\
p &= \text{unit selling price;} \\
c &= \text{unit purchase cost.}
\end{aligned}
$$

Here, the physical holding cost rate h_p refers to the costs related to storing and maintaining inventory, insurances, shrinkage, etc. average out total units sold over a certain period of time. That is, h_p is the cost rate that does *not* include the financial opportunity cost due to holding inventory. The physical backorder cost rate b_p should be viewed the same way – it is the tangible, monetary penalty costs related to backlogging, e.g., expediting delivery costs.

We count the time forward, i.e., $t = 1, 2, ..., t, t+1, ...T$, where T is the end of the horizon. Let D_t is the demand occurred in period t. To examine the system dynamics, we introduce the following state variables:

$$
\begin{aligned}
x_t &= \text{net inventory level at the beginning of period } t; \\
w_t^{'} &= \text{net cash level at the beginning of period } t; \\
w_t &= \text{working capital at the beginning of period } t \ = w_t^{'} + cx_t.
\end{aligned}
$$

Let y_t is the inventory position after ordering, and the order quantity is $(y_t - x_t)$. Define the inventory-related cost and the cash-related gain in period t:

$$
\begin{aligned}
G_t(y_t) &= \mathsf{E}[h_p(y_t - D_t)^+ + b_p(y - D_t)^-], \\
R_t(y_t) &= r(w_t^{'} - c(y_t - x_t))^+ - d(w_t^{'} - c(y_t - x_t))^-,
\end{aligned}
$$

where $x^+ = \max\{x, 0\}$, $x^- = -\min\{x, 0\}$, and $\mathsf{E}[\cdot]$ is the expected value over the random demand. The first term $(y_t - D_t)^+$ in the G_t function is the on-hand inventory at the end of the period, whereas the second term $(y_t - D_t)^-$ is the backorder level. The G_t function represents the inventory holding and backorder cost, or the inventory-related cost in short in period t. Similarly, the term $(w_t^{'} - c(y_t - x_t))$ in the R_t function is the net cash level after inventory payment. It yields an interest gain r if positive and an interest loss d if negative (representing the borrowing rate from financial institutions in order to pay the ordered inventory). The R_t function is the cash-related gain in period t.

The transitions of the inventory and cash states between periods are as follows:

$$
\begin{aligned}
x_{t+1} &= y_t - D_t, &(2.1)\\
w_{t+1}^{'} &= w_t^{'} - c(y_t - x_t) + pD_t + R_t(y_t) - G_t(y_t) \\
&= (1+r)(w_t^{'} - c(y_t - x_t))^+ - (1+d)(w_t^{'} - c(y_t - x_t))^- \\
&\quad +pD_t - G_t(y_t). &(2.2)
\end{aligned}
$$

Equation (2.2) states that the next period's net cash is the result of total cash inflows and outflows of the current period. Here, we assume that the customer will pay on order so the revenue is pD_t. If there is debt, i.e., $(w_t^{'} - c(y_t - x_t))$ is negative, it will incur an interest loss and this debt will carry over to the next period.

The net working capital is

$$
\begin{aligned}
w_{t+1} &= (1+r)(w_t - cy_t)^+ - (1+d)(w_t - cy_t)^- + pD_t - G_t(y_t) + c(y_t - D_t) \\
&= (1+r)w_t + (p-c)D_t - rcy_t - G_t(y_t) - (d-r)(w_t - cy_t)^- &(2.3)\\
&= w_t + (p-c)D_t - G_t(y_t) + r(w_t - cy_t)^+ - d(w_t - cy_t)^-. &(2.4)
\end{aligned}
$$

So far, we have considered a very practical and general environment for a firm that can invest and borrow with different rates.

2.1 Perfect Financial Markets

Let's consider a special case in which the financial market is *perfect*.[1] In our model, this is equivalent to assuming that $r = d$, i.e., the interest

[1] Modigliani and Miller (1958) show that if the financial markets are perfect (no frictions), i.e., absence of taxes, bankruptcy costs, agency costs, asymmetric information, the value of a firm is unaffected by how that firm is financed. This is often referred to *the capital structure irrelevance principle*. Notice that the value of the firm is often created by the operations activities. In other words, this principle

return rate is equal to the borrowing rate. In this case, the firm can freely borrow cash so cash is no longer a concern. This is exactly what the classic inventory model assumes. With this assumption, Equation (2.4) becomes

$$\begin{aligned}
& (1+r)w_t + (p-c)D_t - rcy_t - G_t(y_t) \\
= \quad & (p - c(1+r))D_t + (1+r)w_t \\
& -(h_p + rc)(y_t - D_t)^+ - (b_p - rc)(y_t - D_t)^-. \qquad (2.5)
\end{aligned}$$

Notice that D_t is an exogenous random variable, and w_t is the initial system state at the beginning of period t. Thus, to maximize the expected working capital in period $t+1$, one only needs to minimize the expected cost in Equation (2.5), i.e.,

$$\mathsf{E}[(h_p + rc)(y_t - D_t)^+ + (b_p - rc)(y_t - D_t)^-]. \qquad (2.6)$$

Equation (2.6) is the single-period inventory-related cost in in the classic inventory model. We have a sound economic meaning for the cost parameters. The holding cost rate h is $(h_p + rc)$, which is the sum of the physical holding cost rate h_p and the opportunity cost of capital rc due to inventory investment. The backorder cost rate b is $(b_p - rc)$, which is the physical backorder cost minus the opportunity cost of capital rc.[2] It can be shown that a base-stock policy is optimal. More specifically, one can view the holding cost rate h and backorder cost rate b as follows:

$$\begin{aligned}
h &= h_p + rc; \\
b &= b_p - rc.
\end{aligned}$$

If the demand is i.i.d, the optimal base-stock level s^* can be obtained from the well-known optimality equation as shown in the inventory teaching note: Finding s^* such that

$$P(D \leq s^*) = \left(\frac{b_p - rc}{b_p + h_p}\right) = \left(\frac{b}{b+h}\right).$$

basically suggests that operations decisions may be decoupled from finance decisions. Unfortunately, it is not possible that the markets are of no frictions. In our inventory model, the friction comes from the fact that d is larger than r as the financial institutions have to make a non-zero profit.

[2] It can be shown that the discount rate α is equal to $1/(1+r)$.

For simplicity, let's write

$$s^* = F^{-1}\left(\frac{b}{b+h}\right),$$

where F is the cdf of the demand distribution and F^{-1} is the inverse cdf function.[3]

We pose a discussion on estimating the cost parameters in practice here. According to the above analysis, when the financial market is perfect, the inventory holding cost rate is composed of the physical holding cost rate h_p and the financial holding cost rate rc. Recall that r is the interest rate due to the cash investment in the capital market. Broadly speaking, if a firm conducts investments by financing through debt and equity, the required expected return would be WACC, the weighted average cost of capital. In most OM textbooks, WACC is often recommended to estimate the financial holding cost rate. To estimate the physical holding cost rate, however, is a tough task. The physical holding cost comes from, for example, managing and maintenance expenses, storage, insurances, shrinkage, obsolescence, etc. The list is very long and often business specific. Technically speaking, one should sum up all these costs in a time period and allocate the cost to each inventory unit sold during this period. There are a number of alternative cost accounting systems that can be relevant for some purposes while being inadequate for others. Thus, it is neither always possible nor economical to keep track of all costs, or to split them and allocate them properly. Fortunately, the physical holding cost rate is often small.

As for the backorder rate, this is the penalty cost incurred for an arriving demand that cannot be satisfied immediately due to stock out. Some physical (tangible) penalty costs may occur, e.g., expedited shipping cost, overtime production, etc. Let b_p denote the physical backorder cost. Then, the backorder cost rate is $b = b_p - rc$. This is because a unit of inventory shortage implies that additional cash c was invested in the capital market, gaining the return rate r. Thus, the actual tangible backorder cost rate is b, which is less than b_p. Nevertheless, one should know the backorder cost is beyond the tangible costs. There

[3] If lead time is positive, say L, the demand should be replaced with the total demand during lead time and the review period.

are many intangible costs involved, such as loss of goodwill, market shrinkage, etc.

2.2 Imperfect Financial Markets

So far, we have explained how the inventory model as well as its corresponding cost parameters are formulated when the financial market is perfect, i.e., $d = r$. Let's turn to a more realistic scenario where the financial market is not perfect, i.e., $d > r$. In this case, Equation (2.4) suggests that to maximize the expected working capital in period $t+1$, one has to minimize $\mathsf{E}[G_t(y_t)] - R_t(y_t)$, or equivalently,

$$\begin{aligned}&\mathsf{E}[h_p(y_t - D_t)^+ + b_p(y_t - D_t)^-] - r(w_t' - c(y_t - x_t))^+ \\ &+d(w_t' - c(y_t - x_t))^-.\end{aligned} \tag{2.7}$$

One can see that the inventory decision y_t is affected by the cash level w_t' through the last two terms of Equation (2.7). Thus, the inventory decision cannot be decoupled from the financial flow.

The optimal inventory policy no longer has a simple structure. Luo and Shang (Forthcoming) show that a base-stock policy that depends on the working capital level is near-optimal. More specifically, when demand is i.i.d, define

$$\begin{aligned}\underline{s} &= F^{-1}\left(\frac{b_p - dc}{b_p + h_p}\right) = F^{-1}\left(\frac{b_p - dc}{b + h}\right), \\ s &= F^{-1}\left(\frac{b_p - rc}{b_p + h_p}\right) = F^{-1}\left(\frac{b}{b + h}\right).\end{aligned}$$

Clearly, $\underline{s} \leq s$. The optimal policy is executed as follows. When the working capital level w_t is greater (less, respectively) than cs ($c\underline{s}$, respectively), one should order up to the base stock level s ($\underline{s}$, respectively). If the working capital is between cs and $c\underline{s}$, one should order up to the total working capital level w_t. We refer to this optimal policy as the $(\underline{s}, s)$ policy.

3

Game Description

The pedagogical objective of the Cash Beer Game is to let students understand the challenge of managing inventory and cash simultaneously, and how cash flow influences the inventory order decision. To that end, we also observe the resulting bullwhip effect.

The Cash Beer Game is played similarly as the Beer Game except by adding two activities for cash transfers. The sequence of events is as below.

1. Observe the incoming order (from a downstream firm) or demand (from consumers).
2. Attempt to fill the order (including outstanding backorders, if any) from inventory.
3. Receive cash from a downstream firm for the ordered inventory in Step 1 (a retailer receives cash from consumer's demand).
4. Record remaining inventory or backorders, and calculate inventory holding cost (if net inventory is positive) or backorder cost (if net inventory is negative).
5. Receive a shipment from upstream to inventory.

6. Advance a shipment the upper stream supplier one position downstream.

7. Transmit the previous round's order to the supplier.

8. Pay an upstream firm for the previous round's inventory order to the supplier.

9. Determine this round's order quantity.

Step 3 and Step 8 are new steps compared to the Beer Game. These two steps together essentially move cash from downstream to upstream, which is the same direction as the information flow. In Step 3, the amount of cash received in the current period is equal to the incoming order quantity times the unit selling price; in Step 8. the amount of cash payment to the supplier is equal to the order quantity transmitted in Step 7 times unit purchase cost. In other words, the payment scheme used in this game is pay-at-order. Each player still makes an inventory order decision in Step 9 and this is the only decision that a player has to make in each round. In each round, the inventory-related costs and cash-related costs are incurred. More specifically, the inventory holding cost and backorder cost are calculated according to the net inventory level in Step 4; that is, the inventory *physical* holding cost rate (h_p per case of beer per round) times the number of cases of beer in inventory, and the *physical* backorder cost rate (b_p per case of beer per round) times the number of outstanding backorders. The cash related cost includes an interest gain (negative cost) if the cash level at the end of the round is positive and a borrowing cost if the cash level is negative. The cash level at the end of a round is equal to the initial cash level plus a downstream player's payment received in Step 3 minus the physical inventory holding and backorder costs in Step 4 minus the inventory payment in Step 8. Note that we do not assume that cash is a hard constraint that restricts the inventory order quantity. This allows a player to continue the game even if there is a deficit, but the cash deficit will incur a penalty cost, i.e., the borrowing rate times the negative cash level at the end of a round. This assumes that some financial institutions are willing to lend short-term funds to the firms.

During the game, the net inventory level and cash level are automatically calculated and each player only needs to make an inventory order decision in Step 9 by trading off the physical holding cost rate h_p versus the physical backorder cost rate b_p, and the borrowing rate d versus the interest gain rate r. For the latter tradeoff, this is due to the fact that the inventory order decision will affect the payment to the supplier, which, in turn, affects the cash level. A player attempts to clear the outstanding debt as soon as possible as the borrowing rate d is larger than the interest return rate r. We also assume that the return from selling the product is higher than that of cash return rate, so a player has a motivation to purchase the inventory.

Table 3.1: Parameters for Each Player

	Retailer	Wholesaler	Distributor	Factory
w_1'	40	30	25	20
p	6	3.85	2.75	2.2
c	3.85	2.75	2.2	2
h_p	1.00	0.95	0.67	0.3
b_p	5.00	3.55	2.33	1.2
d	0.5	0.4	0.3	0.2
r	0.25	0.2	0.15	0.1

Table 3.1 shows the parameters of each player. These parameters are chosen to make a player choose the same base-stock level when the borrowing rate d is equal to the interest return rate r if the player optimizes her decision based on the base-stock policy. To see this, take Wholesaler as an example. The physical holding cost rate $h_p = 0.95$ per unit per round and the physical backorder cost $b_p = 3.55$ per unit per round. The financial holding cost rate $= r \times c = 0.2 \times 2.75 = 0.55$ per unit per round. Thus, the holding cost rate $h = h_p + rc = 0.95 + 0.55 = 1.5$ and the backorder cost rate $b = b_p - rc = 3.55 - 0.55 = 3$. The critical ratio is then equal to $b/(b + h) = 3/4.5$, which is the same as that of Wholesaler in the Beer Game, i.e., $1/(1 + 0.5)$. Based on the near-optimal policy derived in Luo and Shang (Forthcoming), the average order quantity in the Cash Beer Game should be smaller than that of

the Beer Game, meaning the bullwhip effect should be dampened. Here, we refer to the “order" bullwhip effect. The “material" bullwhip effect – the variability of shipments – may behave differently; see Chen *et al.* (2017).

The interested reader can visit the website “cashbeergame.com” for more information. The trial and the instruction of the game can be requested from me.

4

Pedagogical Experience

I have developed two online simulation games: Beer Game and Cash Beer Game (visit: cashbeergame.com). These two games are played in a core course of a master of science program at Fuqua School of Business, Duke University. The course has a module that discusses inventory management. In this module, I first discussed a one-time inventory decision model, i.e., the newsvendor model. Then, I introduced these two games in a single class session before I introduced finite-horizon models, i.e., the EOQ model, the base-stock model, and the model with cash flows. There is a teaching note to provide an overview of both simulation games in the course pack.

These two games were played in a class of two hours and fifteen minutes. I first explained the game rule of the Beer Game, followed by the students playing the game. I ran the game for about thirty rounds. Then, I showed the ranking of all teams and demonstrated the bullwhip effect by showing their team work. The bullwhip effect is a well-established topic and the debrief is fairly standard. An instructor usually asks for students' experience, reveals the demand information, explains the causes of the bullwhip effect and discusses the mitigation strategies. The Beer Game takes about one hour to play.

After students returned from a 15-minute break, I started the Cash Beer Game. I first explained the game rule and the focus is on the cash flows as well as the new system state – the net cash level. I did not spend too much time on how the inventory order decision affects the inventory-related and cash-related costs, as well as their tradeoffs. This is one of the objectives that I hope students can figure out the tradeoffs during the game and how operations decision will influence cash and inventory simultaneously. The explanation of the game is about 10 minutes, followed by a game time of 30 minutes (30 rounds). The debrief is about 20 minutes.

In the subsequent class session, I formally introduce the base-stock model. In particular, I discuss how the holding cost rate and backorder cost rate are derived when the financial market is perfect in the standard inventory model. Then, I move the discussion to the scenario in which the financial market is not perfect and introduce an effective control policy. The goal is to emphasize the importance of considering a firm's working capital when making inventory decisions. The lecture is based on the material in Section 2 of this document.

In addition to the above approach of focusing on the optimal policy, the instructor can focus on comparing the bullwhip effect beween two games for the same team. That is, students can be divided into two groups, one playing the Beer Game and the other the Cash Beer Game. The instructor can discuss the bullwhip effect from the Beer Game group and examine the impact of cash flows on the bullwhip effect from the other. A careful design of the lecture may lead to an interesting empirical study.

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