

Coordination Mechanisms in Decentralized Serial Inventory Systems with Batch Ordering

Kevin H. Shang

Fuqua School of Business, Duke University, Durham, North Carolina 27708,
khshang@duke.edu

Jing-Sheng Song

Fuqua School of Business, Duke University, Durham, North Carolina 27708; and
Antai College of Economics & Management, Shanghai Jiao Tong University,
200052 Shanghai, China, jssong@duke.edu

Paul H. Zipkin

Fuqua School of Business, Duke University, Durham, North Carolina 27708,
paul.zipkin@duke.edu

This paper studies a periodic-review, serial supply chain in which materials are ordered and shipped according to (R, nQ) policies. Three information scenarios are considered, depending on the level of information available: echelon, local, and quasilocal. In the echelon scenario, each stage can access the inventory and cost information within its echelon (comprising the stage itself and all downstream stages); in the local scenario, each stage only accesses its own local information. Finally, in the quasilocal scenario, each stage knows its local information, plus the actual customer demands. We propose coordination schemes that regulate the stages to achieve the supply chain's optimal cost under each information setting. All these coordination schemes fit comfortably within an emerging practice called supply chain finance, which includes the organization and technology needed to implement them.

Key words: coordination schemes; batch ordering; demand information; supply chain finance

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1. Introduction

This paper considers supply chains with two complicating features: diverse incentives and bulk shipments. It remains a fact of logistics life that materials often move in large batches. The evidence is plain to see—huge planes, trucks, trains, ships, warehouses, and container yards. Also, supply chains are intricate webs, involving tens or hundreds of organizations, each with its own objectives. Globalization and outsourcing strengthen both features, leading to longer shipping distances and more interorganizational transfers.

For example, consider a supply chain that sells some product in the U.S. market. A local retailer orders from a regional distributor, who further orders from an overseas manufacturer. The product is ordered and shipped in batches (containers, pallets, or cases). Each batch incurs a fixed cost. There are production and/or transportation lead times between the three locations, and each location incurs inventory holding costs. The costs and physical distances are often unevenly distributed throughout the system. They may thus induce some firms to use large batches, even though the overall system would be better off with smaller ones. The question is how to design an incentive scheme to coordinate the decisions of all these firms.

One requirement for coordinating geographically dispersed supply chain partners is information. In practice, companies and networks of companies have different levels of information integration. Advanced information systems, such as enterprise resources planning (ERP), have made systemwide information possible. This rich information may be used to derive a solution that achieves optimal systemwide performance. However, implementing this centralized solution may not be easy. As Griffin and Scherrer (2000, p. 766) write, "One of the criticisms of current ERP implementations is the fact that they are complex and inflexible. As a result, there has been interest in the development of decentralized strategies for enterprise systems. . . . [T]he incentives of the decentralized functions must be aligned based on enterprise goals." In addition, not all firms can install those sophisticated, expensive information systems, so the decision makers may only have access to less complete information. For these reasons, we are interested in designing incentive schemes for different levels of information.

We consider a periodic-review, series inventory system with N stages. Random customer demand occurs at stage 1, stage 1 obtains inventory from stage 2, stage 2 from stage 3, etc., and stage N is replenished by an outside source. Each stage orders an integer

multiple of a base order quantity (or batch size). These base quantities satisfy integer-ratio relations, that is, the base quantity at each stage is an integer multiple of that of the next downstream stage. Unsatisfied demands are backlogged. There are fixed costs for each batch and linear holding and backorder costs. (In some cases, it may be more realistic to posit that each *shipment* incurs a cost, regardless of the number of batches in the shipment. This cost structure is more complex, and we do not consider it here. Anyway, our assumption certainly describes many situations. Many transportation companies charge per tank or per container.)

We consider three information scenarios, representing different levels of information integration: local, echelon, and quasilocal. In the local scenario, each stage accesses only local information, specifically, its own inventory and cost information. We call a stage a *local enterprise* (LE) in this case and denote it $LE(j)$, $1 \leq j \leq N$. Each $LE(j)$ views the order placed by $LE(j-1)$ as its local demand and uses a local-stock (R, nQ) policy to replenish inventories. In the echelon scenario, each stage knows the inventory and cost information of its entire echelon (the stage itself and all downstream stages), including customer demands. Here, we call a stage an *echelon enterprise* (EE) and denote it $EE(j)$. Each EE uses an echelon-stock (R, nQ) policy. In the quasilocal scenario, all customer-demand information is shared by the supply chain partners, but not cost information. In other words, each stage knows its local information, plus the actual customer demands. We call a stage a *quasilocal enterprise* (QE) and denote it $QE(j)$. Information systems that support this level of integration are becoming more common. We introduce a new policy, called a quasilocal-stock (R, nQ) policy, which uses such information. Our objective is to design an effective coordination scheme for each of the three scenarios.

The schemes we propose have a common structure: The firms in the supply chain create, appoint, or hire an *integrator*. If the entire chain belongs to a single company, the integrator can be the owner itself. For a chain composed of several independent firms, the integrator can be one of these firms, a team of them, or a third-party organization. The integrator is responsible for payment transfers between the enterprises, has access to all the available information, and in particular knows the systemwide optimal solution for each information setting. Based on the solution, the integrator designs a contract with three cost terms, namely, a holding cost rate, a backorder cost rate, and a fixed order cost, for each enterprise. In each period, the integrator first compensates all enterprises for their actual costs. Then, each enterprise pays the integrator based on the contract cost terms. With this payment-transfer scheme, each enterprise determines its policy

parameters, aiming to minimize its own average total cost, determined by the contract instead of the actual costs.

We show that, in the echelon scenario, such a scheme can induce each EE to choose the optimal echelon-stock (R, nQ) policy, and thus achieve the systemwide optimal cost. We also develop an effective heuristic scheme in which the contract terms can be obtained directly from the system cost parameters without knowing the optimal solution (see Appendix A). For the local scenario, we devise a scheme that induces the LEs to choose the best local-stock (R, nQ) policies. For the quasilocal scenario, we show that, for any given quasilocal-stock (R, nQ) policy, there exists an equivalent echelon-stock (R, nQ) policy. We then develop an incentive scheme that can achieve the optimal systemwide optimal cost.

These contracts have the flexibility to achieve any allocation of the supply chain's cost among the firms. By setting the parameters appropriately, the integrator can ensure that every party is better off than before. Thus, there is always a contract that Pareto dominates any other policy.

These contracts are also remarkably simple: they specify only three cost parameters—a holding cost rate, a backorder cost rate, and a fixed order cost, for each enterprise. Thus, to provide incentives for the enterprises to work together toward systemwide optimality, all we need to do is to reassign system costs among different players by specifying these parameters. In addition, this three-parameter cost structure has exactly the same form as that for a single-location inventory system. Once these parameters are specified, the optimal order quantity can be readily determined. Therefore, the mechanisms are not only incentive compatible but also easy to execute. Finally, the formulas we derived for these cost parameters provide flexibility to adjust the cost that each enterprise has to pay. This enables coordination of players with different bargaining powers. (See Remark 2 in §3.)

Our coordination schemes fit comfortably within an emerging practice called *supply chain finance* (SCF), which aims to improve the performance of a supply chain by integrating material flows with financial flows. Because a supply chain comprises self-interested parties, it is often necessary to introduce a middleman, an SCF service provider, who governs the payment transfers between supply chain parties based on operational events, such as departures and arrivals of shipments. The SCF provider can be a financial institution (e.g., Bank of America, HSBC), a third- or fourth-party logistics provider (e.g., United Parcel Service, Kuehne + Nagel), a software company (e.g., Orbian, SAP), a specialized supply chain coordinator (e.g., Li and Fung), or a combination thereof. One challenge for SCF is to design financial

metrics and transaction policies that can align operational decisions between the supply chain parties. Our coordination schemes can be seen as prototypes of such alignment. We refer the reader to Aberdeen Group (2006; 2008a, b), Demica (2007), and SAP (2005) for further discussions of SCF.

The study of centralized control for multiechelon inventory systems was initiated by Clark and Scarf (1960, 1962). They show that an echelon base-stock policy is optimal for the system without fixed order costs. For the system with fixed costs, they point out that an optimal policy, if exists, may be complex and hard to implement. Therefore, researchers have focused on evaluating simple policies, such as (R, nQ) policies. See, for example, Axsäter (1993a), Chen and Zheng (1994), and Shang and Song (2007). Chen (1998, 2000) develops algorithms for finding optimal echelon-stock and local-stock reorder points with fixed base quantities. DeBodt and Graves (1985) analyze an approximate cost model. Chen and Zheng (1998) develop an algorithm to search for an optimal echelon-stock (R, nQ) policy. Because the exact algorithm is complicated, they also provide heuristics. Shang (2008) develops a near-optimal heuristic for base quantities, which solves N single-stage (R, nQ) systems. Shang et al. (2008) provides an approach to obtain the optimal batch sizes for the local-stock (R, nQ) policy.

More recently, several authors have studied coordination of decentralized systems, but only without fixed order costs. Most of the models assume base-stock control. For the echelon scenario, Watson and Zheng (2005) propose an incentive scheme where each stage is measured based on its echelon accounting inventory level. For the local scenario, Chen (1999), Lee and Whang (1999), and Porteus (2000) propose different schemes, such that the participants' local decisions achieve the centralized solution. Cachon and Zipkin (1999) treat both the local and echelon scenarios. These results are possible because there is a one-to-one correspondence between echelon base-stock policies and local ones. Here, however, this is not so. Local (R, nQ) policies are special cases of echelon ones (Axsäter and Rosling 1993).

The rest of this paper is organized as follows. Section 2 introduces the model and reviews prior results. Section 3 presents the echelon-based scheme. Section 4 discusses the local scenario. Section 5 introduces the quasilocal-stock (R, nQ) policy. Section 6 discusses implementation and summarizes this paper. Appendix A describes an echelon-based heuristic scheme. Appendix B derives the steady-state distribution of downstream orders received by each LE.

2. Preliminaries

We consider a periodic-review inventory system with N stages. Customer demand occurs at stage 1. Stage 1

obtains supplies from stage 2, stage 2 from stage 3, etc., and stage N is replenished by an outside source with ample supply. Demands in different periods are independent, identically distributed, nonnegative, and integer valued. Let μ denote the mean one-period demand. Unsatisfied demand is backlogged. In each period, the echelon holding cost h_j is incurred for each unit held in echelon j , and backorder cost b is incurred for each unit of backorders at stage 1. Define $h'_j = \sum_{i=j}^N h_i$, the local holding cost for stage j . There is a fixed cost k_j for each base quantity ordered by stage j . (For example, an order including three base quantities costs $3k_j$.) The transportation lead time L_j between stage $j+1$ and stage j is constant. The base order quantities satisfy integer-ratio relations $Q_j = q_j Q_{j-1}$, $j = 1, 2, \dots, N$, where $Q_0 = 1$, and q_j is a positive integer.

The sequence of events in each period is as follows: At the start of the period, each stage (1) receives an order from stage $j-1$; (2) places an order to stage $j+1$; (3) receives a shipment sent from stage $j+1$; and (4) sends a shipment to stage $j-1$. (Stage 1 skips steps 1 and 4, whereas stage N orders from the outside source.) Stage 1 decides its order first, followed by stage 2, and so on, until stage N . The shipments are made in the opposite order, starting at stage N , then stage $N-1$, etc., until stage 1. After orders and shipments, demand occurs during the period. Inventory holding and backorder costs are assessed at the end of the period.

In the echelon scenario, each stage j implements an echelon-stock (R, nQ) policy: At the beginning of each period t , $t = 0, 1, 2, \dots$, the echelon inventory order position $IOP_j(t)$ (stage j 's outstanding orders + stage j 's on-hand inventory + inventories at or in transit to stages $i < j$ – backorders) is reviewed. If this quantity is less than or equal to the echelon reorder point R_j , the stage orders an integer multiple of Q_j units so as to raise the inventory order position above R_j but no more than $R_j + Q_j$. Similarly, in the local scenario, each stage uses a local-stock (R, nQ) policy with parameters (r_j, Q_j) . At the beginning of each period t , the local inventory order position $IOP'_j(t)$ (outstanding orders + on-hand inventory – backorders from the immediate downstream stage) is reviewed, and the stage orders an integer multiple of Q_j units to raise the inventory order position above r_j but no more than $r_j + Q_j$. The quasilocal-stock scenario uses similar logic but a different state variable, described later.

We assume that the system starts with nonnegative local inventory order positions, i.e., $IOP'_j(0) \geq 0$, for all j . In the local scenario, we choose the local reorder point r_j such that

$$IOP'_j(0) - r_j = l_j Q_{j-1}, \quad (1)$$

where $0 < l_j \leq q_j$ is an integer (Axsäter and Rosling 1993). Recall that orders both to and from stage j are

integer multiples of Q_{j-1} . Thus, changing an r_j that satisfies (1) to $r'_j = r_j + a_j$ for $0 < a_j < Q_{j-1}$ does not affect the timing of future orders. We also assume the following:

ASSUMPTION 1 (CHEN AND ZHENG 1994). $IOP'_j(0)$ is an integer multiple of Q_{j-1} .

This assumption is plausible to avoid carrying extra inventory. Thus, (1) and Assumption 1 together imply that the local reorder point r_j is an integer multiple of Q_{j-1} .

Axsäter and Rosling (1993) provide conditions for a local-stock (R, nQ) policy, $(r_j, Q_j)_{j=1}^N$, and an echelon-stock (R, nQ) policy, $(R_j, Q_j)_{j=1}^N$, to be equivalent (that is, to generate the same ordering decisions). The two are equivalent if and only if

$$R_1 = r_1, \quad R_j = r_j + \sum_{i=1}^{j-1} (r_i + Q_i). \quad (2)$$

Thus, for any local-stock (R, nQ) policy there exists an equivalent echelon-stock (R, nQ) policy, but *not* vice versa, because r_j must be an integer multiple of Q_{j-1} . (If Assumption 1 is not required, r_j needs to satisfy (1).)

Chen and Zheng (1994) develop a recursive procedure to evaluate the total cost of an echelon-stock (R, nQ) policy under Assumption 1. Let $D[t]$ and $D[t]$ denote the demands over $t+1$ and t periods. Define

$$G_1(y) = E[h_1(y - D[L_1]) + (b + h'_1)(y - D[L_1])^-]. \quad (3)$$

For $j = 2, \dots, N$,

$$G_j(y) = E[h_j(y - D[L_j]) + G_{j-1}(O_{j-1}[y - D[L_j]])], \quad (4)$$

where

$$O_j[x] = \begin{cases} x & x \leq R_j + Q_j, \\ x - nQ_j & \text{otherwise.} \end{cases} \quad (5)$$

Here, n is the smallest integer such that $x - nQ_j \leq R_j + Q_j$. Denote $\mathbf{R}, \mathbf{Q} = (R_j, Q_j)_{j=1}^N$, the average total cost per period is

$$C(\mathbf{R}, \mathbf{Q}) = \sum_{i=1}^N \frac{k_i \mu}{Q_i} + \frac{\sum_{x=1}^{Q_N} G_N(R_N + x)}{Q_N}. \quad (6)$$

To evaluate a local-stock (R, nQ) policy under Assumption 1, we can convert it to an equivalent echelon one and use the recursion above.

Let us now turn to optimization. For $N = 1$, the problem of minimizing (6) reduces to

$$\min_{R_1, Q_1} C(R_1, Q_1) = \frac{k_1 \mu + \sum_{x=1}^{Q_1} G(R_1 + x)}{Q_1},$$

where $G(y) = E[h'_1(y - D[L_1]) + (b + h'_1)(y - D[L_1])^-]$. We omit the subscript 1 and approximate demand

and the inventory position as continuous to facilitate the discussion. Specifically, the inventory order position is uniformly distributed over $(R, R + Q]$, and the approximate problem becomes

$$\min_{R, Q} C(R, Q) = \frac{k\mu + \int_0^Q G(R + x) dx}{Q}.$$

Zheng (1992) derives the optimality conditions for this single-stage system. Specifically, it can be shown that $C(R, Q)$ is jointly convex. The optimal solution satisfies the following conditions:

$$C(R^*, Q^*) = G(R^*) = G(R^* + Q^*), \quad (7)$$

$$k\mu = Q^* G(R^*) - \int_0^{Q^*} G(R^* + y) dy. \quad (8)$$

(A discrete version of the optimality conditions can be derived similarly; see Chen and Zheng 1998.)

For $N > 1$, the best echelon-stock policy (R_j^*, Q_j^*) can be found by using the algorithms developed by Chen and Zheng (1998) and Shang and Zhou (2006). Denote the resulting optimal cost by C_E^* . In §3, we shall propose an incentive scheme for the decentralized system to achieve C_E^* . As for the local-stock (R, nQ) policy, Shang et al. (2008) recently provided an approach to find the best base quantities. Denote the best local base quantities as Q_j^ℓ . With these base quantities, the optimal local reorder points r_j^* can be found by using the algorithm developed by Chen (1998). Let C_L^* denote the cost of (r_j^*, Q_j^ℓ) . In §4, we propose an incentive scheme for the decentralized supply chain that induces each LE to choose this policy.

To simplify the discussion and exposition, we approximate the demand and the inventory positions as continuous variables in the rest of this paper.

3. Echelon Mechanism

This section describes the echelon scheme. We assume that the integrator possesses full system information, and in particular knows the optimal solution (R_j^*, Q_j^*) , $j = 1, \dots, N$. In each period, there are money transfers between the integrator and each enterprise in the following sequence. First, at the end of each period, the integrator compensates all EEs for their *actual* costs. Specifically, the integrator pays $EE(j)$, $j \geq 1$, h_j per unit of echelon on-hand inventory I_j , and $EE(1)$ b per unit of backorders B . Also, he pays $EE(j)$, $j \geq 1$, k_j for each base quantity ordered.

Next, at the end of each period t , the integrator charges each $EE(j)$ according to its *accounting* echelon inventory level $IOP_j(t - \tilde{L}_j) - D[t - \tilde{L}_j, t]$, where $\tilde{L}_j = \sum_{i=1}^j L_i$. Here, $IOP_j(t - \tilde{L}_j)$ is $EE(j)$'s echelon inventory order position after placing an order at the beginning of period $t - \tilde{L}_j$, and $D[t - \tilde{L}_j, t]$ is the total demand in periods $t - \tilde{L}_j, t - \tilde{L}_j + 1, \dots, t$. The contract

specifies three cost parameters, (h_j^e, b_j^e, k_j^e) . At the end of each period, EE(j) pays the integrator the inventory backorder costs based on the accounting echelon inventory level, using the specified holding cost rate h_j^e and backorder cost rate b_j^e . In addition, EE(j) pays the integrator the fixed order cost k_j^e for each base quantity ordered.

The objective for EE(j) is to minimize its average total cost per period under this scheme. Because the integrator compensates EE(j)'s actual costs, EE(j) only needs to consider the average cost incurred by implementing the contract. Under the contract, the expected inventory backorder cost given $IOP_j(t) = y$ is

$$G_j^e(y) = E[h_j^e(y - D[\tilde{L}_j]) + (h_j^e + b_j^e)(y - D[\tilde{L}_j])^-].$$

Because y is uniformly distributed over $(R_j, R_j + Q_j]$ (see Zipkin 1986a), the total average cost for EE(j) is

$$\frac{k_j^e \mu + \int_0^{Q_j} G_j^e(R_j + x) dx}{Q_j}.$$

Consequently, our goal is to determine (h_j^e, b_j^e, k_j^e) such that EE(j) will choose (R_j^*, Q_j^*) . We say such parameters “coordinate the system.” For this purpose we can use the optimality conditions (7) and (8). Let \tilde{F}_j denote the cumulative distribution function (cdf) of $D[\tilde{L}_j]$, and \tilde{F}_j^1 the loss function of $\tilde{D}[\tilde{L}_j]$, i.e., $\tilde{F}_j^1(x) = \int_x^\infty (1 - \tilde{F}_j(y)) dy$.

PROPOSITION 1. *The incentive scheme with cost parameters (h_j^e, b_j^e, k_j^e) below coordinates the serial system:*

$$\begin{aligned} h_j^e &= \theta_j h_j, \\ b_j^e &= h_j^e \left[\frac{Q_j^*}{\tilde{F}_j^1(R_j^*) - \tilde{F}_j^1(R_j^* + Q_j^*)} - 1 \right], \\ k_j^e &= \frac{1}{\mu} \left[Q_j^* G_j^e(R_j^*) - \int_0^{Q_j^*} G_j^e(R_j^* + y) dy \right], \end{aligned}$$

where θ_j is any positive real number, $j = 1, \dots, N$.

PROOF. Set $h_j^e = \theta_j h_j$. From (7), (R_j^*, Q_j^*) satisfies $G_j^e(R_j^*) = G_j^e(R_j^* + Q_j^*)$. That is,

$$\begin{aligned} E[h_j^e(R_j^* - D[\tilde{L}_j]) + (h_j^e + b_j^e)(R_j^* - D[\tilde{L}_j])^-] \\ = E[h_j^e(R_j^* + Q_j^* - D[\tilde{L}_j]) + (h_j^e + b_j^e)(R_j^* + Q_j^* - D[\tilde{L}_j])^-]. \end{aligned}$$

Equivalently,

$$\begin{aligned} h_j^e R_j^* - h_j^e E[D[\tilde{L}_j]] + (h_j^e + b_j^e) \tilde{F}_j^1(R_j^*) \\ = h_j^e (R_j^* + Q_j^*) - h_j^e E[D[\tilde{L}_j]] + (h_j^e + b_j^e) \tilde{F}_j^1(R_j^* + Q_j^*). \end{aligned}$$

The solution to this equation is b_j^e above. Similarly, from (8), we have

$$\frac{k_j^e \mu + \int_0^{Q_j^*} G_j^e(R_j^* + y) dy}{Q_j^*} = G_j^e(R_j^*).$$

The solution is k_j^e above. \square

Under this scheme, EE(j)'s optimal decision (R_j^*, Q_j^*) is independent of the other EEs' decisions. Thus, the coordinated solution is a Nash equilibrium. Furthermore, from (7), the EE(j)'s average total cost is

$$\begin{aligned} G_j^e(R_j^*) &= \theta_j E[h_j(R_j^* - D[\tilde{L}_j]) \\ &\quad + (h_j + (b_j^e/\theta_j)(R_j^* - D[\tilde{L}_j])^-)]. \end{aligned} \quad (9)$$

So far, we have demonstrated that the contract (h_j^e, b_j^e, k_j^e) coordinates the supply chain. Below we demonstrate that the proposed scheme is implementable or incentive-compatible. That is, each EE and the integrator will be better off after implementing this scheme.

EXAMPLE 1. Consider the example in the second paragraph of the introduction. There are three stages. EE(1), EE(2), and EE(3) describe the retailer, the regional distributor, and the overseas manufacturer, respectively. Assume the following parameters: $(L_1, L_2, L_3) = (1, 5, 2)$, $(k_1, k_2, k_3) = (30, 100, 10)$, $(h_1, h_2, h_3) = (1, 0.25, 0.1)$, and $b = 9$. Demand is a Poisson process with rate $\lambda = 4$ (so $\mu = 4$). Initially, the EEs use the following policies: $(R_1, Q_1) = (4, 14)$, $(R_2, Q_2) = (24, 28)$, and $(R_3, Q_3) = (32, 28)$. With these policies, the expected cost per period incurred by EE(1) is $(k_1 \mu / Q_1) + h_1 E[I_1] + b E[B] = 24.04$. Similarly, the expected cost per period for EE(2) is $(k_2 \mu / Q_2) + h_2 E[I_2] = 17.96$ and for EE(3) is $(k_3 \mu / Q_3) + h_3 E[I_3] = 5.01$.¹

With the systemwide information, the integrator can obtain the optimal policy, which is $(R_1^*, Q_1^*) = (7, 16)$, $(R_2^*, Q_2^*) = (28, 48)$, and $(R_3^*, Q_3^*) = (36, 48)$. According to Proposition 1, the integrator specifies the following cost terms for the EEs, parameterized by θ_j :

$$(h_1^e, b_1^e, k_1^e) = \theta_1(1, 8.62, 23.41), \quad (10)$$

$$(h_2^e, b_2^e, k_2^e) = \theta_2(0.25, 5.45, 61.52), \quad (11)$$

$$(h_3^e, b_3^e, k_3^e) = \theta_3(0.1, 1.91, 24.07). \quad (12)$$

If EE(j) accepts these contract terms, EE(j) would choose (R_j^*, Q_j^*) and expect to pay the integrator $G_j^e(R_j^*)$ per period, where

$$(G_1^e(R_1^*), G_2^e(R_2^*), G_3^e(R_3^*)) = (15\theta_1, 12\theta_2, 4.8\theta_3).$$

Thus, EE(j) will agree to the scheme only if he pays less under the new contract, that is,

$$15\theta_1 < 24.04, \quad 12\theta_2 < 17.96, \quad 4.8\theta_3 < 5.01. \quad (13)$$

On the other hand, the integrator will agree to the scheme only if he can make a profit. The integrator is expected to receive $15\theta_1 + 12\theta_2 + 4.8\theta_3$ per period from

¹ Note that $I_j = IL_j + B$ and $B = \max\{0, -IL_1\}$, where IL_j is the echelon inventory level for stage j (see, e.g., Shang and Song 2007). The distribution of IL_j can be computed from a recursion in Chen and Zheng (1994).

EEs. Suppose that each EE implements the optimal policy (R_j^*, Q_j^*) ; the integrator is expected to pay the optimal cost $C_E^* = 38.68$. In other words, the integrator will participate this scheme if

$$15\theta_1 + 12\theta_2 + 4.8\theta_3 > C_E^* = 38.68. \quad (14)$$

Note that the sum of the right-hand sides of (13) is 47.01, which is greater than 38.68, the right-hand side of (14). So, there exist solutions to the system of inequalities (13) and (14). Hence, the scheme is implementable. Once the θ_j s are selected, the contract cost terms can be determined by (10)–(12). For example, one solution is $(\theta_1, \theta_2, \theta_3) = (1.4, 1.2, 1)$, leading to $(h_1^e, b_1^e, k_1^e) = (1.4, 12.07, 32.77)$, $(h_2^e, b_2^e, k_2^e) = (0.3, 6.54, 73.82)$, $(h_3^e, b_3^e, k_3^e) = (0.1, 1.91, 24.07)$.

REMARK 1. Cachon (2003) points out that the coordinated solution in Chen's (1999) scheme for base-stock policies is a Nash equilibrium. We obtain the same result here for the model with echelon (R, nQ) policies. For the local and quasilocal schemes discussed in §§4 and 5, similar transfer-payment mechanics can be designed. With the same logic, we can show that, in each case, the coordinated solution is a Nash equilibrium, and there exists a coordinating contract that dominates any other policy. Thus, our focus will be on the cost terms in the contracts.

REMARK 2. By adjusting θ_j , our contract provides considerable flexibility in how much each enterprise pays. This feature enables the integrator to coordinate supply chain players with different bargaining powers. For example, if the integrator uses the above values of θ_j , from (13), EE(1), EE(2), and EE(3) should expect to pay 21, 14.4, and 4.8 per period, respectively. This implies cost savings of 3.04, 3.56, and 0.21 per period for EE(1), EE(2), and EE(3), respectively. Consider another scenario where EE(1) has stronger bargaining power and insists on a larger cost saving. In this case, the integrator can select a smaller θ_1 to increase the cost saving for EE(1). For instance, if the integrator uses $(\theta_1, \theta_2, \theta_3) = (1.1, 1.4, 1)$, then EE(1), EE(2), and EE(3) expect to pay 16.5, 16.8, and 4.8 per period, respectively. Now, the cost saving for EE(1) becomes 7.54.

For the case where the base quantities are fixed, the above scheme reduces to a two-parameter scheme with cost terms (h_j^e, b_j^e) . In this case, the order costs are fixed and do not affect the decisions.

COROLLARY 1. *The contract with the cost parameters (h_j^e, b_j^e) coordinates the serial system when base order quantities are fixed.*

It may be hard to compute the optimal solution (R_j^*, Q_j^*) and therefore to obtain the cost factors in the scheme above. In Appendix A, we describe a simple heuristic scheme, whose cost terms can be obtained directly from the original cost parameters without knowing the solution.

4. Local Mechanism

This section considers the local information scenario. Each LE implements a local-stock (R, nQ) policy. A local-stock (R, nQ) policy operates much like an echelon-stock (R, nQ) policy. The local inventory order position (outstanding orders + on-hand inventories – backorders from the immediate downstream stage) is monitored. Recall that stage 1 orders first, followed by stage 2, and so on, until stage N . Define

$IOP_j^-(t)$ = local inventory order position after receiving downstream orders,

$IOP_j^+(t)$ = local inventory order position after placing an order, if necessary.

If $IOP_j^-(t) \leq r_j$, an order of nQ_j units is placed, and $IOP_j^+(t)$ is equal to $IOP_j^-(t) + nQ_j$, where n is the smallest integer such that $IOP_j^+(t) \in \{r_j + 1, r_j + 2, \dots, r_j + Q_j\}$; otherwise, $IOP_j^+(t) = IOP_j^-(t)$.

The best outcome we can achieve from a local-stock (R, nQ) policy is C_L^* . We propose a scheme similar to the one above. Three cost parameters (h_j^e, b_j^e, k_j^e) are specified. LE(j) is evaluated according to its *accounting* local inventory level at the end of each period. Similar to the echelon policy, the order decision at the beginning of period $t - L_j$ determines the accounting local inventory level at the end of period t , that is, $IOP_j^-(t - L_j) - D_j[t - L_j, t]$. Here, $D_j[t - L_j, t]$ is the total orders received by LE(j) in periods $t - L_j, t - L_j + 1, \dots, t$. LE(j) selects its policy parameters (r_j, Q_j) to minimize its average total cost.

"Local" sounds easy and straightforward, but actually it requires more intricate methods than the echelon scenario. First, LE(1)'s inventory order position IOP_1^+ is uniformly distributed over $\{r_1 + 1, r_1 + 2, \dots, r_1 + Q_1\}$ (Zipkin 1986a). Next, LE(2) views LE(1)'s orders as its demands. These form a Markov-modulated demand process, that is, the demand is governed by the inventory order position at LE(1), a Markov chain with states $\{r_1 + 1, r_1 + 2, \dots, r_1 + Q_1\}$ and a uniform stationary distribution. The cumulative demand at LE(2) has nondecreasing sample paths. Because LE(2) employs a local-stock (R, nQ) policy, its local inventory order position is uniformly distributed over $\{r_2 + Q_1, r_2 + 2Q_1, \dots, r_2 + Q_2\}$. Repeating this logic for all j , we conclude that in steady state IOP_j^+ has a uniform distribution over $\{r_j + Q_{j-1}, r_j + 2Q_{j-1}, \dots, r_j + Q_j\}$. Denote by $D_j(T)$ and $D_j[T]$ the demand over T and $T + 1$ periods for LE(j). Our goal is to find costs (h_j^e, b_j^e, k_j^e) that induce LE(j) to select (r_j^*, Q_j^*) to solve the following problem:

$$\min_{r_j, Q_j} \frac{k_j^e \mu + \sum_{x=1}^{Q_j} G_j^e(r_j + x)}{Q_j}, \quad (15)$$

where

$$G_j^\ell(y) = E[h_j^\ell(y - D_j[L_j]) + (h_j^\ell + b_j^\ell)(y - D_j[L_j])^-].$$

(The stages solve their problems in a bottom-up fashion. Start at stage 1, proceed to stage 2, and so on.) The hard part for LE(j), $j > 1$, however, is to estimate $D_j[L_j]$. Appendix B shows how to compute its steady-state distribution.

If we approximate the demand as continuous in (15), following Zipkin (1986b), the objective $(k_j^\ell \mu + \int_0^{Q_j} G_j^\ell(r_j + x) dx)/Q_j$ is jointly convex. Thus, by setting $h_j^\ell = \theta_j h_j'$, $\theta_j > 0$, the optimality conditions in (7) and (8) can be applied to find the desired parameters (b_j^ℓ, k_j^ℓ) with given (r_j^*, Q_j^ℓ) . Let F_j denote the cdf of the lead-time demand for LE(j), i.e., $F_j(y) = P(D_j[L_j] \leq y)$, and F_j^1 the loss function of $D_j[L_j]$, i.e., $F_j^1(x) = \int_x^\infty (1 - F_j(y)) dy$.

PROPOSITION 2. *The incentive scheme with the cost parameters $(h_j^\ell, b_j^\ell, k_j^\ell)$ coordinates the serial system with local-stock (R, nQ) policies, where*

$$\begin{aligned} h_j^\ell &= h_j' \theta_j, \\ b_j^\ell &= h_j^\ell \left[\frac{Q_j^\ell}{F_j^1(r_j^*) - F_j^1(r_j^* + Q_j^\ell)} - 1 \right], \\ k_j^\ell &= \frac{1}{\mu} \left[Q_j^\ell G_j^\ell(r_j^*) - \int_0^{Q_j^\ell} G_j^\ell(r_j^* + y) dy \right]. \end{aligned}$$

PROOF. The proof is similar to that of Proposition 1, and was thus omitted. \square

In the case of fixed base quantities, the three-parameter contract reduces to a two-parameter contract, using (h_j^ℓ, b_j^ℓ) only.

COROLLARY 2. *The contract with cost parameters (h_j^ℓ, b_j^ℓ) coordinates the system with fixed base order quantities.*

5. Quasilocal Mechanism

Recall that in the quasilocal information scenario, we assume that every QE learns the customer demands as they occur, either from the integrator or from the immediate downstream stage. We introduce a new inventory control policy, the quasilocal (R, nQ) policy, which augments the local-inventory information with demand information. We show that, for any given quasilocal-stock (R, nQ) policy, there exists an equivalent echelon-stock (R, nQ) policy. Then, we design an incentive scheme that induces the QEs to achieve C_E^* .

To describe the quasilocal-stock policy, we introduce a quantity called the *virtual* inventory order position (VIP). The VIP is an inventory position that reflects the local inventory status after demand occurs. Simply speaking, $VIP = \text{outstanding orders} + \text{local on-hand inventories} - \text{customer demand}$. Under

this policy, each QE controls the material flow according to two parameters, (r_j^q, Q_j^q) . Below we describe the inventory dynamics.

Define

$VIP_j^-(t)$ = virtual inventory order position at stage j at the beginning of period t after receiving demand $D[t-1, t]$;

$VIP_j(t)$ = virtual inventory order position at stage j at the beginning of period t after placing an order, if necessary.

At the beginning, set $VIP_j(0) = IOP_j'(0) = I_j'(0)$. Also, let $d_t = D[t, t+1)$ for simplicity. The dynamics of $VIP_j(t)$ and $IOP_j'(t)$, $t = 0, 1, 2, \dots$, are determined by the following three events. We assume that the QEs perform these events sequentially, from QE(1) to QE(2), etc., until QE(N).

(1) Demand d_t occurs. It is known by QE($j+1$), $j = 0, \dots, N-1$ at the beginning of period $t+1$, before placing an order.

(2) Update $VIP_j^-(t+1) = VIP_j(t) - d_t$; update $IOP_j'(t+1) = IOP_j'(t) - D_j[t, t+1)$, where $D_j[t, t+1)$ is QE($j-1$)'s order at the beginning of period $t+1$.

(3) If $VIP_j^-(t+1) \leq r_j^q$, order integer multiples of Q_j^q , i.e., set $VIP_j(t+1) = VIP_j^-(t+1) + nQ_j^q$ such that $VIP_j(t+1) \in \{r_j^q + 1, r_j^q + 2, \dots, r_j^q + Q_j^q\}$. Update $IOP_j'(t+1) = IOP_j'(t+1) + nQ_j^q$. Otherwise, set $VIP_j(t+1) = VIP_j^-(t+1)$ and $IOP_j'(t+1) = IOP_j'(t+1)$.

In short, VIP_j reflects the effective local inventory level after taking into account the demand; IOP_j' is the conventional local inventory order position. The inventory-replenishment rule is similar to the classic local-stock (R, nQ) policy. The only difference is that now the replenishment decision is triggered by VIP_j^- .

The policy parameters (r_j^q, Q_j^q) for the quasilocal-stock policy at QE(j) can be determined from an echelon-stock policy (R_j, Q_j) , with the initial local inventory-order positions satisfying Assumption 1. The parameters (r_j^q, Q_j^q) are

$$Q_j^q = Q_j \quad \text{for all } j, \quad (16)$$

$$r_1^q = R_1, \quad r_j^q = IOP_j'(0) - (IOP_j(0) - R_j). \quad (17)$$

Here is an intuitive explanation for (17): Each r_j^q must satisfy $VIP_j(0) - r_j^q = IOP_j(0) - R_j$ so that both the quasilocal policy and the echelon policy trigger the same orders. Because $VIP_j(0) = IOP_j'(0)$, Equation (17) follows immediately. Given this construction, we can show

PROPOSITION 3. *For any given echelon-stock (R, nQ) policy, $(R_j, Q_j)_{j=1}^N$, there exists an equivalent quasilocal-stock (R, nQ) policy, $(r_j^q, Q_j^q)_{j=1}^N$.*

PROOF. We consider $j = 2$; the result for larger j can be proven similarly. We need to prove two results for all t :

(i) $IOP_1(t) - R_1 = VIP_1(t) - r_1^q$ and $IOP_2(t) - R_2 = VIP_2(t) - r_2^q$;

(ii) $IOP_1(t) = IOP'_1(t)$ and $IOP_2(t) = IOP'_1(t) + IOP'_2(t)$.

Note that (i) ensures these two policies trigger the orders at the same times, and (ii) ensures the policies generate the same echelon inventory order positions.

We first consider $t = 0$. $IOP_1(0) - R_1 = VIP_1(0) - r_1^q$ holds by definition. $IOP_2(0) - R_2 = IOP'_2(0) - r_2^q + IOP'_2(0) - IOP'_2(0) = IOP'_2(0) - r_2^q = VIP_2(0) - r_2^q$. Thus, the result (i) is true. The result (ii) is also true by definition.

Now consider $t = 1$. Suppose the demand in period 0 is d_0 . We first prove (i). For stage 1,

$$\begin{aligned} IOP_1(1) - R_1 &= IOP_1(0) - d_0 + nQ_1 - R_1, \\ &\quad n \text{ some integer } \geq 0 \\ &= VIP_1(0) - d_0 + nQ_1 - R_1 = VIP_1(1) - r_1^q. \end{aligned}$$

For stage 2,

$$\begin{aligned} IOP_2(1) - R_2 &= IOP_2(0) - d_0 + nQ_2 - R_2, \quad n \text{ some integer } \geq 0 \\ &= VIP_2(0) - r_2^q - d_0 + nQ_2 \quad (\text{from (i) for } t = 1) \\ &= VIP_2(1) - r_2^q. \end{aligned}$$

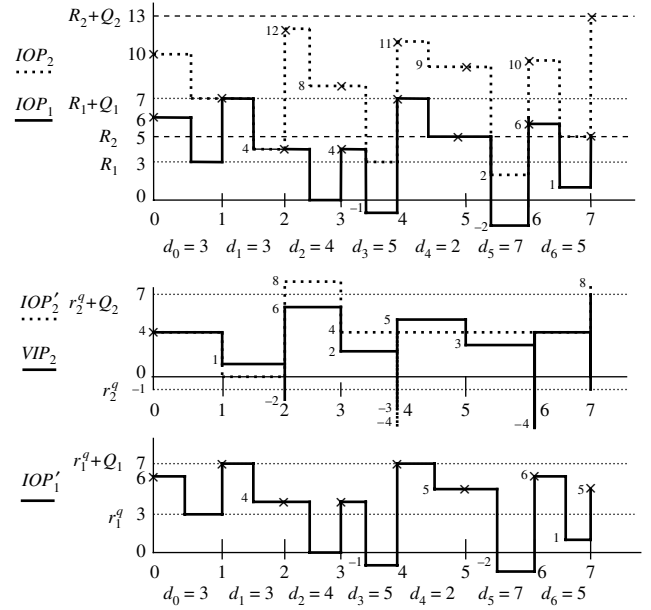
For part (ii), $IOP_1(1) = IOP'_1(1)$ by definition. For stage 2,

$$\begin{aligned} IOP_2(1) &= IOP_2(0) - d_0 + nQ_2, \quad n \text{ some integer } \geq 0 \\ &= IOP'_1(0) + IOP'_2(0) - d_0 + nQ_2 \\ &\quad (\text{from (ii) for } t = 0) \\ &= IOP'_1(0) - d_0 + n'Q_1 + IOP'_2(0) - n'Q_1 + nQ_2, \\ &\quad n' \text{ some integer } \geq 0 \\ &= IOP'_1(1) + IOP'_2(1) + nQ_2 \\ &= IOP'_1(1) + IOP'_2(1). \end{aligned}$$

Thus, both (i) and (ii) hold for $t = 1$. A simple induction along these lines proves the result for all t . \square

Figure 1 illustrates IOP_j , IOP'_j , and VIP_j for a two-stage system in the first seven periods. It shows a sample path with $d_0 = d_1 = 3$, $d_2 = 4$, $d_3 = 5$, $d_4 = 2$, $d_5 = 7$, and $d_6 = 5$. The echelon-stock policies are $(R_1, Q_1) = (3, 4)$ and $(R_2, Q_2) = (5, 8)$. The initial echelon inventories are $I_1(0) = 6$ and $I_2(0) = 10$. The corresponding local-stock parameters are $(r_1^q, Q_1^q) = (3, 4)$ and $(r_2^q, Q_2^q) = (4 - (10 - 5), 8) = (-1, 8)$. The top chart shows the dynamics of the echelon inventory order positions over time for stages 1 and 2. The middle and bottom charts represent the dynamics of the local inventory order position and the virtual inventory order position. It is clear that $IOP'_1 = VIP_1$. But for

Figure 1 Echelon Inventory Order Positions, Virtual Inventory Order Positions, and Local Inventory Order Positions for a Two-Stage Example



stage 2, these two curves depart from each other. It is interesting to see that stage 2 places an order of 8 units at $t = 2$, although stage 1 does not place an order. This is because the replenishment decision is triggered by $VIP_2^-(2) = -2$, which is smaller than the reorder point $r_2^q = -1$. After the order is placed, $VIP_2(2) = -2 + 8 = 6$, and $IOP'_2(2) = 0 + 8 = 8$. Thus, the local inventory order position IOP'_j may not fall in the interval $(r_j^q, r_j^q + Q_j^q)$. Also, the echelon inventory order positions IOP_j are the same at the beginning of each period for the echelon-stock (R, nQ) policy and the quasilocal-stock (R, nQ) policy.

From the optimal echelon-stock policy (R_j^*, Q_j^*) and the initial inventory order positions, the corresponding quasilocal-stock policy parameters (r_j^{q*}, Q_j^{q*}) can be derived from (16) and (17). We now introduce an incentive scheme that induces $QE(j)$ to select (r_j^{q*}, Q_j^{q*}) .

The incentive scheme is similar to the one in §4. Three cost parameters (h_j^*, b_j^*, k_j^*) are specified for each j . Each $QE(j)$ is charged according to the virtual inventory level, which is equal to $VIP_j(t - L_j) - D[t - L_j, t]$. A fixed cost k_j^* is charged for each order. From $QE(j)$'s perspective, the demand determines the dynamics of VIP_j . Thus, VIP_j is uniformly distributed over $\{r_j^q + 1, r_j^q + 2, \dots, r_j^q + Q_j^q\}$ in steady state. The problem for $QE(j)$ is

$$\min_{r, Q} \frac{k_j^* \mu + \sum_{x=1}^{Q_j^q} G_j^q(r + x)}{Q_j^q},$$

where

$$G_j^q(y) = \mathbb{E}[h_j^*(y - D[L_j]) + (b_j^* + k_j^*)(y - D[L_j])^-].$$

Table 1 Required Information and the Best Cost Achieved by the Proposed Incentive Schemes

Incentive scheme	Required information	Best cost
Echelon scheme: for EE(j)	$h_i, L_i, 1 \leq i \leq j, b_j^e, k_j^e$, customer demands	C_E^*
Local scheme: for LE(j)	h_j', L_j, b_j^e, k_j^e , downstream orders	C_L^*
Quasiloca scheme: for QE(j)	h_j', L_j, b_j^*, k_j^* , customer demands	C_E^*

Again, by applying (7) and (8), we can obtain the cost parameters in the contract. Let $F_{(j)}$ denote the cdf of $D[L_j]$ and $F_{(j)}^1$ denote the loss function of $D[L_j]$, i.e., $F_{(j)}^1(x) = \int_x^\infty (1 - F_{(j)}(y)) dy$. The proposition below summarizes the result, under a continuous approximation of demand.

PROPOSITION 4. *The incentive scheme with cost parameters (h_j^*, b_j^*, k_j^*) coordinates the serial system with quasiloca-stock (R, nQ) policies, where*

$$\begin{aligned} h_j^* &= \theta_j h_j', \\ b_j^* &= h_j' \left[\frac{Q_j^{q*}}{F_{(j)}^1(r_j^{q*}) - F_{(j)}^1(r_j^{q*} + Q_j^{q*})} - 1 \right], \\ k_j^* &= \frac{1}{\mu} \left[Q_j^{q*} G_j^q(r_j^{q*}) - \int_0^{Q_j^{q*}} G_j^q(r_j^{q*} + y) dy \right], \end{aligned}$$

where $\theta_j > 0$. The average systemwide cost is C_E^* .

Observe that, under the proposed scheme, a QE does not need to know the costs and lead times of downstream stages. This is an advantage over the echelon scheme, when cost data are proprietary for supply chain partners.

Table 1 provides a summary of the required information for each EE, LE, or QE to determine its policy parameters and the best cost that can be achieved by the proposed incentive schemes.

REMARK 3. Our virtual inventory position is the same as that of Axsäter (1993b), Graves (1996), and Axsäter et al. (2002). Both reflect the inventory levels that are committed to demands at the most downstream stage. Theirs is used to determine an allocation rule in a distribution system, whereas ours drives the local order policy.

6. Concluding Remarks

Most previous studies on coordination mechanisms in supply chains focus on systems with no fixed order costs and consequently base-stock policies. Here, we design mechanisms for systems with fixed costs and, therefore, batch ordering. Such costs represent the fixed costs of production setups and/or scale economies in transportation. They are often unevenly

distributed throughout the system. They may thus induce some participants to use large batches, even though the overall system would be better off with smaller ones. We find that by redistributing these costs in a careful but simple way, all the players can be induced to use batch sizes as well as safety stocks that work well for the system as a whole. This case requires careful attention to the information available to each enterprise in the chain.

Specifically, we analyze a serial inventory system, where each stage implements an echelon-stock, a local-stock, or a quasiloca-stock (R, nQ) policy, and each has its own objective. We develop coordination schemes based on cost parameters derived from optimal solutions. For the echelon scenario, we describe a scheme that achieves the true minimum long-run average cost. We also present a simple, effective heuristic scheme. For the local scenario, we present a scheme that can induce the stages to choose the best local-stock (R, nQ) policy. Finally, we present a new information scenario, quasiloca, that augments the local inventory information with demand information. With this measure, a similar incentive scheme induces the system to achieve the optimal systemwide cost. The quasiloca scheme is particularly useful when cost data are proprietary to supply chain partners.

These coordination schemes fit comfortably within the emerging practice of supply chain finance. They allow any cost allocation among the enterprises. It is always possible to find an allocation that leaves every enterprise no worse off than before. Thus, each enterprise will have an incentive to participate in the arrangement.

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Appendix A. An Echelon Heuristic Scheme

We propose a heuristic scheme. The objective is to generate reasonable costs $(\hat{h}_j^e, \hat{b}_j^e, \hat{k}_j^e)$ to replace (h_j^e, b_j^e, k_j^e) without knowing the optimal solution.

We first briefly review a simple heuristic developed by Shang (2008). He shows how to compute near-optimal base quantities in two steps, clustering and minimization. In the clustering step, the stages are grouped into disjoint clusters $\{c(1), c(2), \dots, c(M)\}$ according to cost ratios. Define

$$h[m] = \sum_{i \in c(m)} h_i, \quad h'[m] = \sum_{i \in c(m)} h_i', \quad \text{and} \quad k[m] = \sum_{i \in c(m)} k_i.$$

These clusters satisfy the following two conditions:

- $k[1]/h[1] < \dots < k[M]/h[M]$, and
- for each cluster $c(m) = \{l_1, \dots, l_2\}$, there does not exist an l with $l_1 \leq l < l_2$ so that $k[m^-]/h[m^-] \leq k[m^+]/h[m^+]$, where $c(m^-) = \{l_1, \dots, l\}$ and $c(m^+) = \{l+1, \dots, l_2\}$.

This step is exactly the same as the clustering step in the deterministic model of Maxwell and Muckstadt (1985).

In the minimization step, each cluster solves a single-stage (R, nQ) problem by restricting the base quantity for each cluster to an integer multiple of that of the next downstream one. Specifically, let $Q_{c(m)}$ denote the base quantity for cluster $c(m)$, $m = 1, \dots, M$, where $c(m)$ contains stages i , $i \in \{v, v+1, \dots, v+n(m)-1\}$, and $n(m)$ is the number of stages in cluster $c(m)$. That is, $Q_{c(m)}$ solves the following single-stage problem:

$$\min_{R, Q} \frac{\mu k[m] + \int_0^Q G_{c(m)}(R+x) dx}{Q}, \quad (A1)$$

where

$$G_{c(m)}(y) = E[h[m](y - D[\tilde{L}_{v+n(m)-1}]) + (n(m)b + h'[m])(y - D[\tilde{L}_{v+n(m)-1}])^-], \quad (A2)$$

subject to $Q = qQ_{c(m-1)}$, q a positive integer, $m > 1$. The $Q_{c(m)}$ are found recursively, starting with $m = 1$. Then, set $Q_i^s = Q_{c(m)}$ for $i \in c(m)$; these are the heuristic base quantities. The reorder points (R_1^s, \dots, R_N^s) can be found as in Chen (2000). Shang numerically shows that the policy (R_j^s, Q_j^s) is near optimal.

Here, the idea is to design cost terms $(\hat{h}_j^e, \hat{b}_j^e, \hat{k}_j^e)$ which induce $EE(j)$ to choose a policy close to (R_j^s, Q_j^s) . Given $(\hat{h}_j^e, \hat{b}_j^e, \hat{k}_j^e)$, $EE(j)$'s problem is

$$\min_{R, Q} \frac{\hat{k}_j^e \mu + \int_0^Q \hat{G}_j^e(R+x) dx}{Q}, \quad (A3)$$

where

$$\hat{G}_j^e(y) = E[\hat{h}_j^e(y - D[\tilde{L}_j]) + (\hat{h}_j^e + \hat{b}_j^e)(y - D[\tilde{L}_j])^-],$$

and Q is an integer multiple of Q_{j-1} , $j > 1$.

$EE(j)$ solves this problem in two steps. First, find the optimal reorder point $R(Q)$ for fixed Q , then solve for the optimal Q_j^e for (A3):

$$Q_j^e = \arg \min_Q \left\{ \frac{\hat{k}_j^e \mu + \int_0^Q \hat{G}_j^e(R(Q)+x) dx}{Q} \right\}, \quad \text{s.t. } Q = qQ_{j-1} \quad j > 1. \quad (A4)$$

Now, multiply the right-hand side in (A4) by the constant $h[m]/\hat{h}_j^e$. The result is the equivalent problem

$$\min_Q \frac{(h[m]/\hat{h}_j^e)\hat{k}_j^e \mu + \int_0^Q \hat{G}_{c(m)}(R(Q)+x) dx}{Q}, \quad (A5)$$

where

$$\hat{G}_{c(m)}(y) = E \left[h[m](y - D[\tilde{L}_j]) + \left(\left(\frac{h[m]}{\hat{h}_j^e} \right) (\hat{b}_j^e + \hat{h}_j^e) \right) (y - D[\tilde{L}_j])^- \right]. \quad (A6)$$

Problems (A1) and (A5) have the same structure. In particular, if we set

$$\hat{b}_j^e = \frac{(n(m)b + h'[m] - h[m])\hat{h}_j^e}{h[m]} \quad \text{and} \quad \hat{k}_j^e = \frac{k[m]\hat{h}_j^e}{h[m]}, \quad (A7)$$

the two problems are identical except for the lead-time demands in (A2) and (A6), namely, $D[\tilde{L}_{v+n(m)-1}]$ and $D[\tilde{L}_j]$. Below we report a numerical study, showing that this difference has little impact on the difference between Q_j^e and Q_j^s .

Table A.1 Summary of Solution Gap and Cost Performance

	$b = 50$ (512 instances)			$b = 10$ (512 instances)		
	Average	Max.	Exact (%)	Average	Max.	Exact (%)
R_j^e	1.37	6	28.97	2.28	10	25.70
Q_j^e	0.64	12	53.65	0.77	24	47.85
C^e (%)	1.18	6.31	—	2.49	6.80	—

Also, the reorder point R_j^e should be reasonably close to R_j^s . This is because, if $Q_j^s = Q_j^e$ for all j , then R_j^e is a tight upper bound on R_j^s (Shang and Song 2007).

We can use the same approach discussed in §3 to generate the cost parameters $(\hat{h}_j^e, \hat{b}_j^e, \hat{k}_j^e)$. That is, set $\hat{h}_j^e = \theta_j h_j$ for any positive θ_j , and obtain the corresponding \hat{b}_j^e and \hat{k}_j^e from (A7).

We test the heuristic scheme above on a three-stage system with Poisson demand with average demand rate λ . The parameters are $b = 10, 50$, $\lambda = 5$, $h_j = 0.1, 1$, $L_j = 0.5, 2$, $k_j = 10, 100$, for $j = 1, 2, 3$. The total number of instances is 1,024. We conjecture that (R_j^e, Q_j^e) should be close to (R_j^s, Q_j^s) . To test this, define

$$R_j^e = |R_j^s - R_j^e| \quad \text{and} \quad Q_j^e = |Q_j^s - Q_j^e|$$

to represent the gap between these solutions for each stage. We then calculate the average and maximum gap over the three stages and the 1,024 cases. Table A.1 summarizes the results. It reports separately the cases with $b = 10$ (small b) and $b = 50$ (large b). The column "Exact" gives the percentage of the stages with $Q_j^e = Q_j^s$ (or $R_j^e = R_j^s$). We also compare the total cost obtained from the heuristic scheme, C_E^e , with the minimum cost C_E^s . Denote by C^e the percentage gap between these costs, specifically,

$$C^e = \frac{(C_E^e - C_E^s) \times 100}{C_E^s} \%.$$

The last row in Table A.1 reports this quantity. It is clear that the system cost induced by the heuristic is close to optimal. When b is large, the performance seems better. Also, R_j^e and R_j^s (Q_j^e and Q_j^s) are close in most instances. Thus, the heuristic approach works quite well.

Appendix B. The Steady-State Distribution of Downstream Orders

The demand seen by $LE(j)$ is the order process from $LE(j-1)$. These orders are *not* independent over time. We present a recursive procedure to determine the distribution of lead-time demand for all stages.

For $j \geq 2$, let $IOP_{j-1}^c(t) = r_{j-1} + Q_{j-1} - IOP'_{j-1}(t)$. Then, $\{IOP_{j-1}^c(t)\}$ is a Markov chain with state space $\{0, Q_{j-2}, \dots, (q_{j-1}-1)Q_{j-2}\}$ and a uniform steady-state distribution over the state space. Moreover,

$$IOP_{j-1}^c(t+1) = [IOP_{j-1}^c(t) + D_{j-1}[t, t+1]] \bmod(Q_{j-1}).$$

For $n = 0, 1, \dots$, define

$$\begin{aligned} a_{ik}^n &= P(D_j[t, t+1] = nQ_{j-1}, IOP_{j-1}^c(t+1) \\ &= kQ_{j-2} \mid IOP_{j-1}^c(t) = iQ_{j-2}) \end{aligned}$$

$$\begin{aligned}
 &= P(D_{j-1}[t, t+1] = nQ_{j-1} + (k-i)Q_{j-2}) \\
 &= P\left(D[t, t+1] = n\left(\prod_{\ell=2}^{j-1} q_{\ell}\right)Q_1 + (k-i)Q_{j-2}\right),
 \end{aligned}$$

where $\prod_{\ell=u}^v q_{\ell} = 1$ is $u > v$. Then,

$$\begin{aligned}
 P(D_j[t, t+L_j+2] = nQ_{j-1}, IOP_{j-1}^c(t+L_j+2) = kQ_{j-2}) \\
 = \sum_{i=0}^{q_{j-1}-1} \sum_{m=0}^n \{P(D_j[t, t+L_j+1] = mQ_{j-1}, \\
 IOP_{j-1}^c(t+L_j+1) = iQ_{j-2})a_{ik}^{n-m}\}. \quad (B1)
 \end{aligned}$$

Assuming $IOP_{j-1}^c(t)$ has the steady-state uniform distribution, the initial step is

$$\begin{aligned}
 P(D_j[t, t+1] = nQ_{j-1}, IOP_{j-1}^c(t+1) = kQ_{j-2}) \\
 = \sum_{i=0}^{q_{j-1}-1} a_{ik}^n / q_{j-1}.
 \end{aligned}$$

Summing (B1) over k , we obtain the distribution of $D_j[L_j]$.

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